The expected longest common substring, the Chvátal-Sankoff constant, and

1975

Markov decision processes

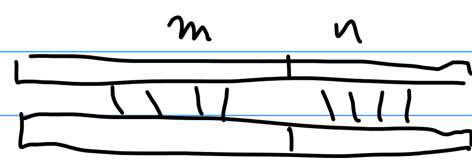
$$\mathrm{LCS}\begin{pmatrix}\text{Banane}\\ \text{anfangen}\end{pmatrix} = \cancel{4}\ 5$$

$$T_n = E\left[\mathrm{LCS}\begin{pmatrix}x_1 x_2 \dots x_n\\ y_1 y_2 \dots y_n\end{pmatrix}\right]$$

random bits

$$\boxed{\gamma = \gamma_2 := \lim_{n\to\infty} \frac{T_n}{n}} \approx 0{,}81$$

$$T_{m+n} \geq T_m + T_n$$



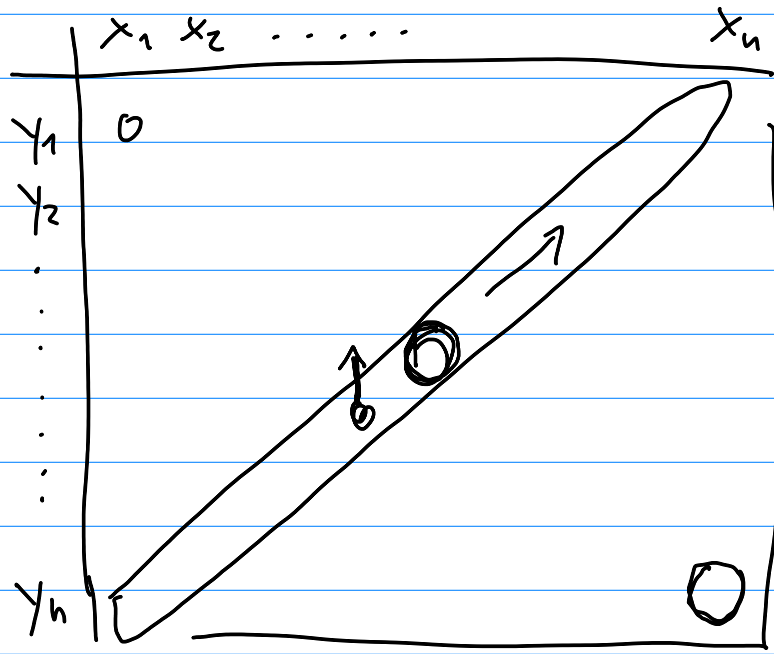

M. Paterson, V. Dančík (1994): $0{,}7739 \leq \gamma \leq 0{,}8377$

G. S. Lueker (1999, SODA 2003, ALENEX 2008, JACM 2009):

$$0{,}788 \leq \gamma \leq 0{,}8263$$

diagonal LCS = DLCS

$$\mathrm{DLCS}\begin{pmatrix} x_1 \cdots\cdots x_n \\ y_1 \cdots\cdots y_n \end{pmatrix} = \max_k \mathrm{LCS}\begin{pmatrix} x_1 \cdots x_k \\ y_1 \cdots\cdots y_{n-k} \end{pmatrix}$$




Alexander (1994):

$$\lim_{n\to\infty} \frac{E\left[\mathrm{DLCS}\begin{pmatrix} x_1\, x_2 \cdots\, x_n \\ y_1\, y_2 \cdots\, y_n \end{pmatrix}\right]}{n} = \frac{\gamma}{2}$$

Window of size $k=6$

**111001010011001010010110100010101010110...**
**1011110010011100101001010101001010001010...**

look at a new bit: $\quad$ cost $= \alpha \approx \delta/2$

find a match: $\quad$ gain $= 1$

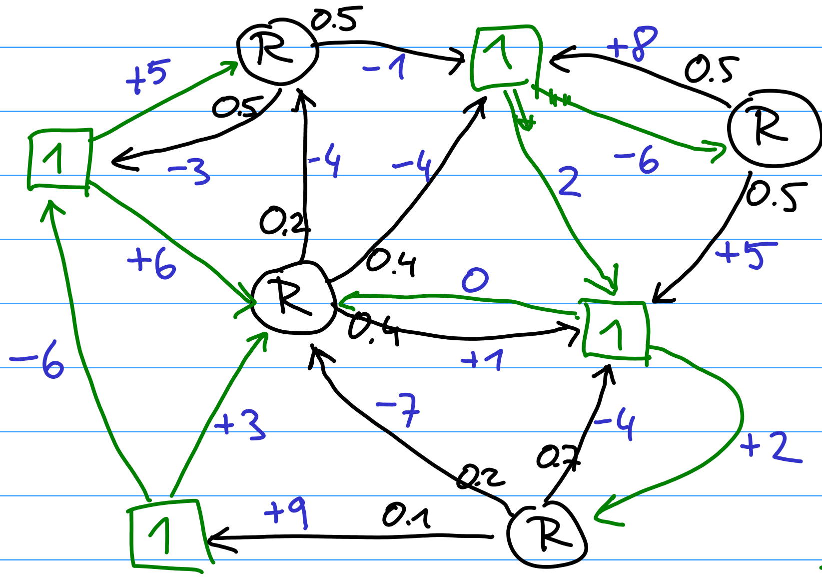
$$\begin{pmatrix} \cancel{1}00101 \\ \cancel{1}10010 \end{pmatrix} \xrightarrow{+1} \begin{pmatrix} 00101 \\ 10010 \end{pmatrix}$$

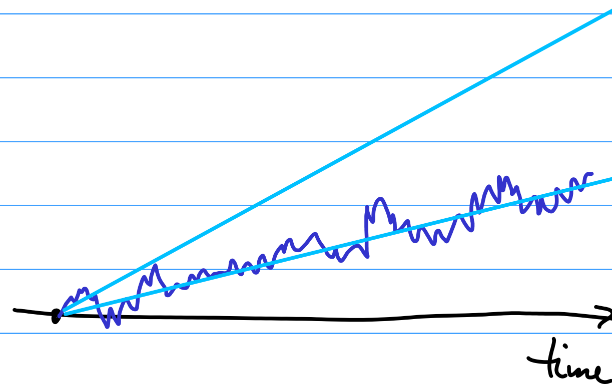
$$\begin{pmatrix} 00101 \\ 10010 \end{pmatrix} \xrightarrow[1/2]{-\alpha} \begin{pmatrix} 001010 \\ 10010 \end{pmatrix} \qquad \begin{pmatrix} 00101 \\ 10010 \end{pmatrix} \xrightarrow[1/2]{-\alpha} \begin{pmatrix} 001011 \\ 10010 \end{pmatrix}$$

$$\begin{pmatrix} 001011 \\ 10010 \end{pmatrix} \xrightarrow[1/2]{-\alpha} \begin{pmatrix} 001011 \\ 100100 \end{pmatrix} \qquad \begin{pmatrix} 001011 \\ 10010 \end{pmatrix} \xrightarrow[1/2]{-\alpha} \begin{pmatrix} 001011 \\ 100101 \end{pmatrix}$$

$$\begin{pmatrix} 001011 \\ 100100 \end{pmatrix} \xrightarrow{\pm 0} \begin{pmatrix} 01011 \\ 100100 \end{pmatrix} \qquad \begin{pmatrix} 001011 \\ 100100 \end{pmatrix} \xrightarrow{\pm 0} \begin{pmatrix} 001011 \\ 00100 \end{pmatrix}$$

# Markov Decision Process (MDP)




positional strategy

$$\boxed{1}\quad v\begin{pmatrix}1a\\1b\end{pmatrix} := v\begin{pmatrix}0a\\0b\end{pmatrix} = 1 + v\begin{pmatrix}a\\b\end{pmatrix} \qquad \left(a,b \in \{0,1\}^*\right)$$

$$\text{(R)}\quad v\begin{pmatrix}a\\b\end{pmatrix} := -\alpha + \frac{1}{2}\left( v\begin{pmatrix}a0\\b\end{pmatrix} + v\begin{pmatrix}a1\\b\end{pmatrix} \right)$$

$$\text{or } -\alpha + \frac{1}{2}\left( v\begin{pmatrix}a\\b0\end{pmatrix} + v\begin{pmatrix}a\\b1\end{pmatrix} \right)$$

$$\boxed{1}\quad v\begin{pmatrix}0a\\1b\end{pmatrix} = v\begin{pmatrix}1b\\0a\end{pmatrix} := \max\left( v\begin{pmatrix}a\\1b\end{pmatrix}, v\begin{pmatrix}0a\\b\end{pmatrix} \right)$$

- If there is a solution $v : \left(\{0,1\}^k \cup \{0,1\}^{k-1}\right)^2 \to \mathbb{R}$ with "$\leq$" then player 1 will not go broke in expectation: $\alpha \leq \gamma/2$
- value iteration: start with $v \equiv 0$
  set $v^{new}(x) := \min\left(v^{old}(x), \text{formula}\right)$ for all $x$