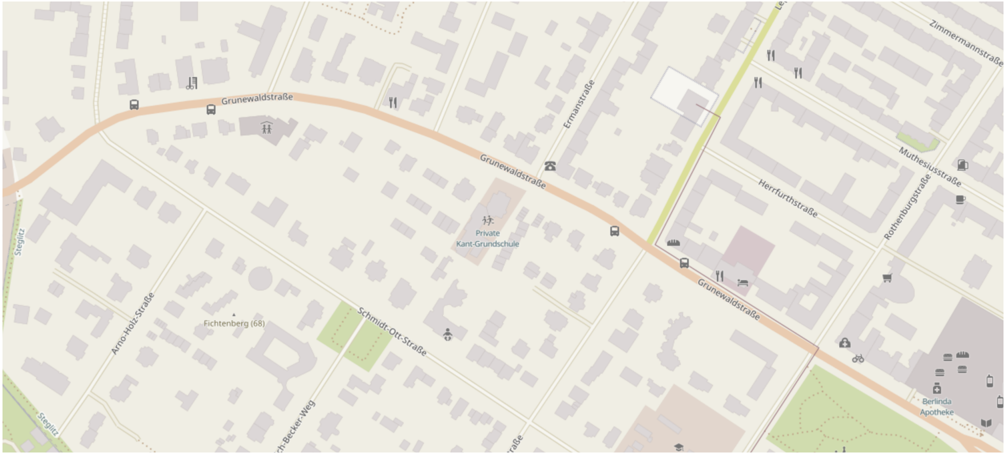
Noon Seminar
On Optimal Polyline Simplification Using the Hausdorff and Fréchet Distance

Alexander Kauer

Paper by van Kreveld, Löffler, Wiratma, JoCG ’20 [3]

2020-05-28

Polyline Simplification



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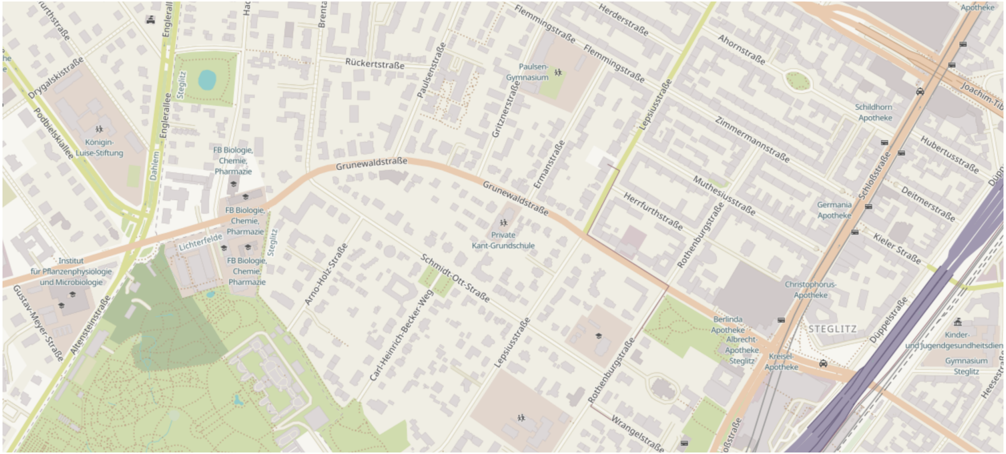

(c) openstreetmap.org contributors, CC BY-SA

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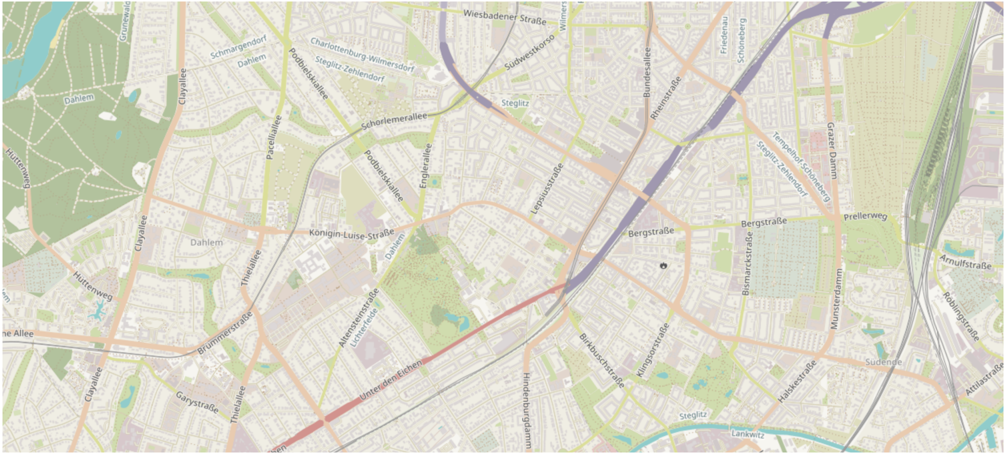

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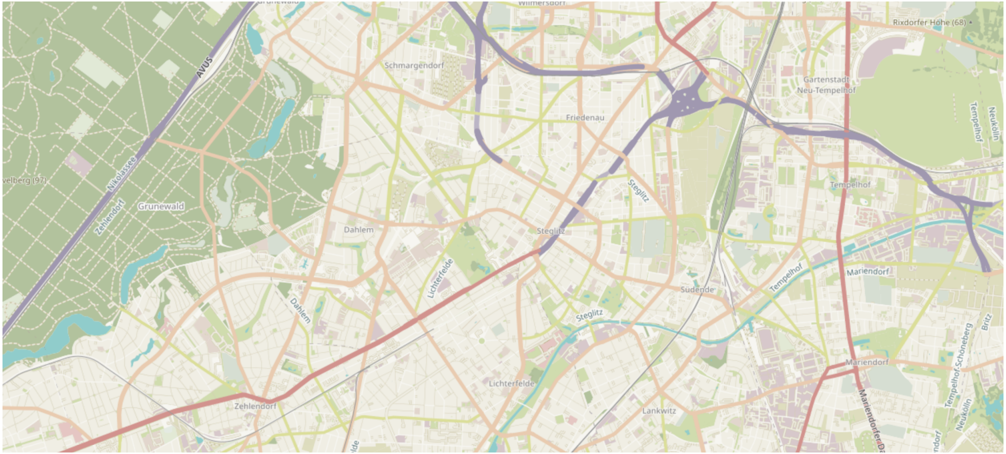

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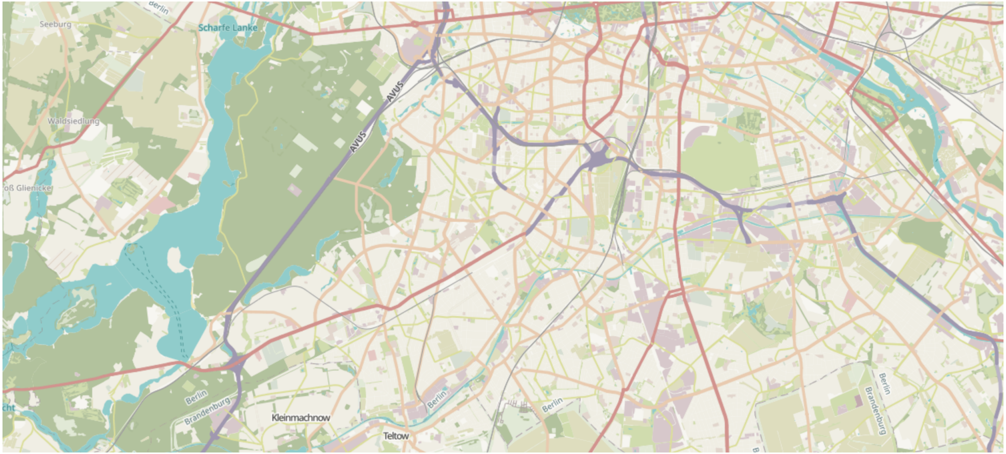

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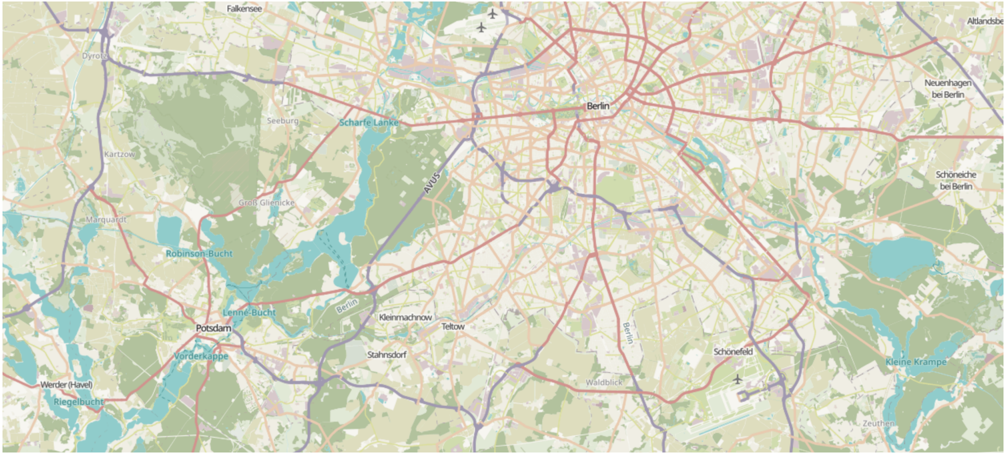
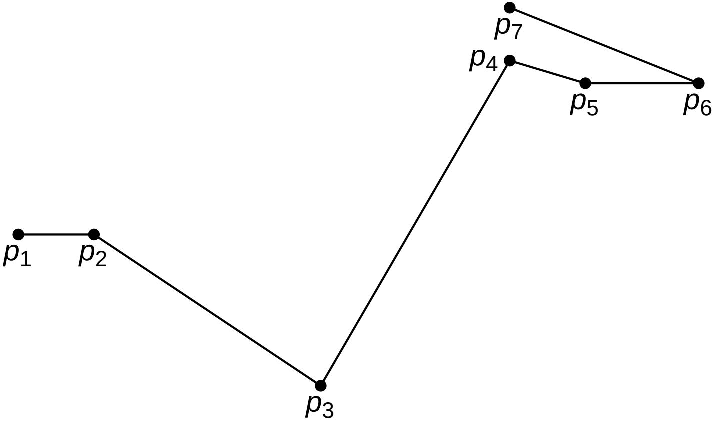


Polyline Simplification

Given a polyline $p_1, \ldots, p_n$. We want to find a minimum size subsequence, starting with p_1 and ending with p_n, which is close to the original polyline.



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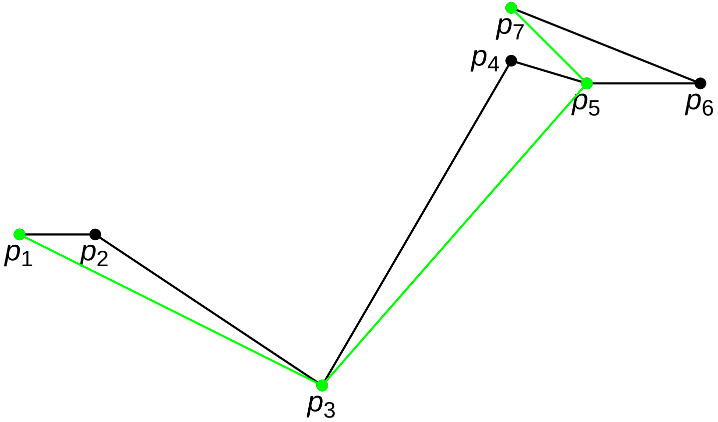

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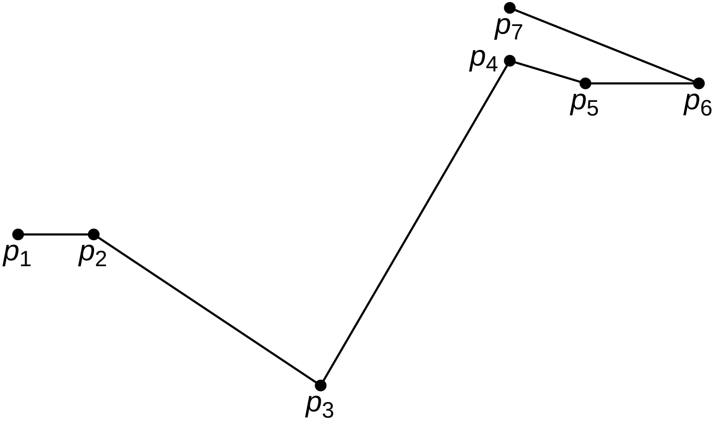
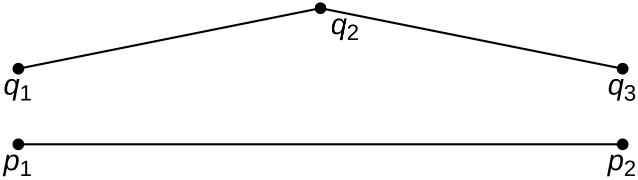


Close: Fréchet distance or Hausdorff distance $\leq \varepsilon$.

Fréchet Distance



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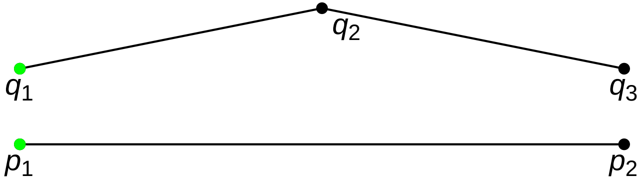

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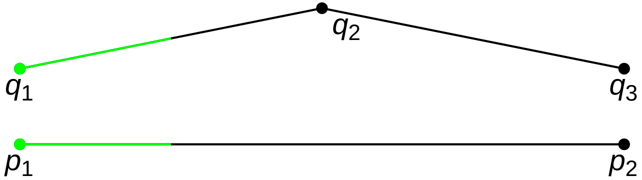

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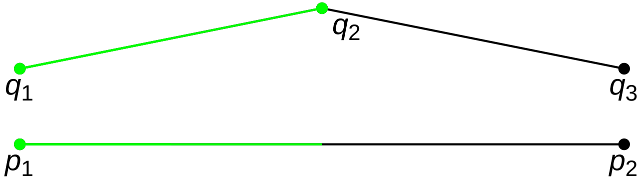

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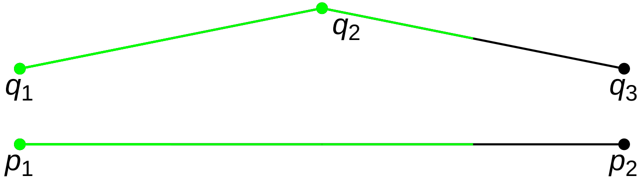

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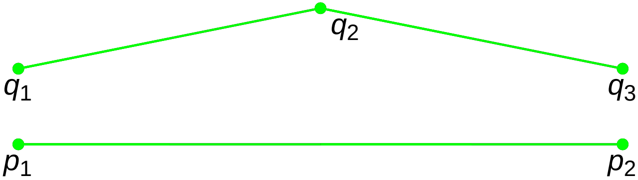

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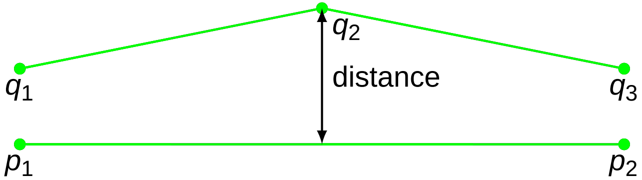
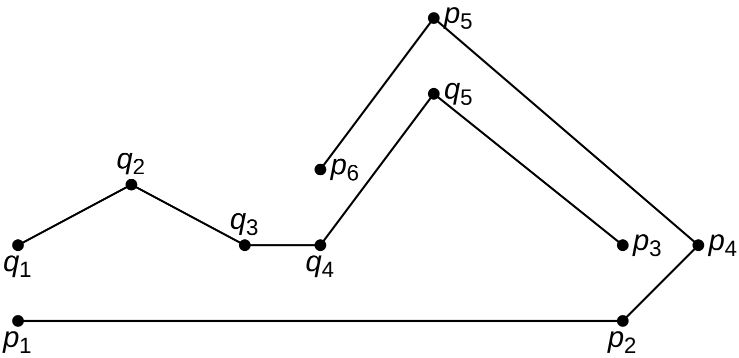


Hausdorff Distance



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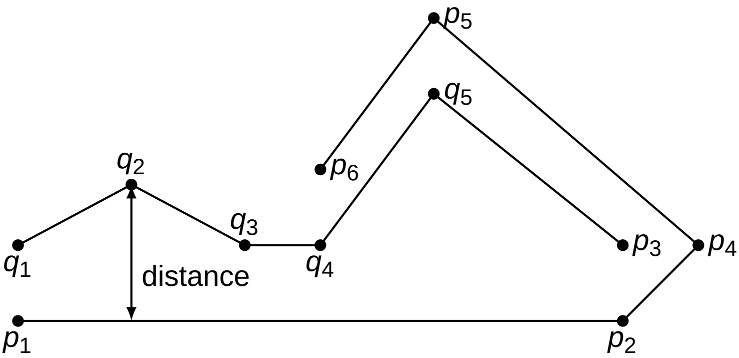
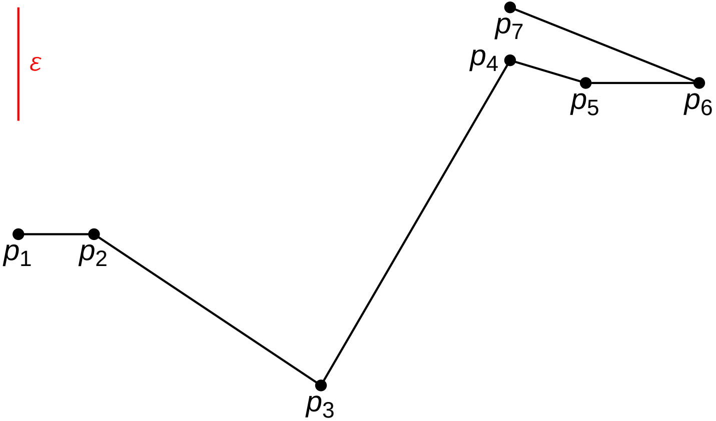
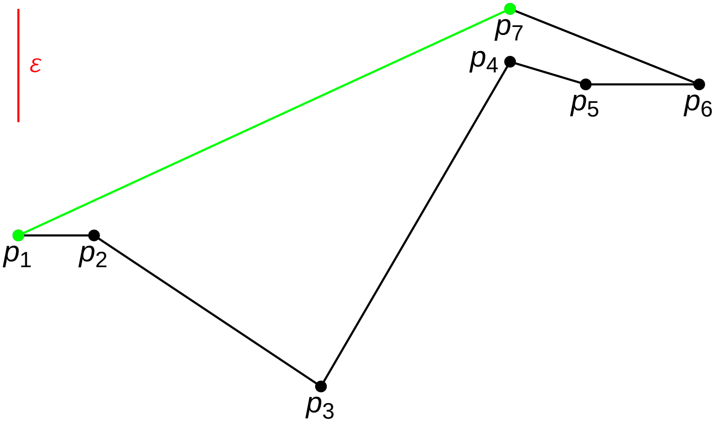


Douglas-Peucker



Douglas-Peucker

- Find farthest point.
- If too far away: subdivide and recurse.



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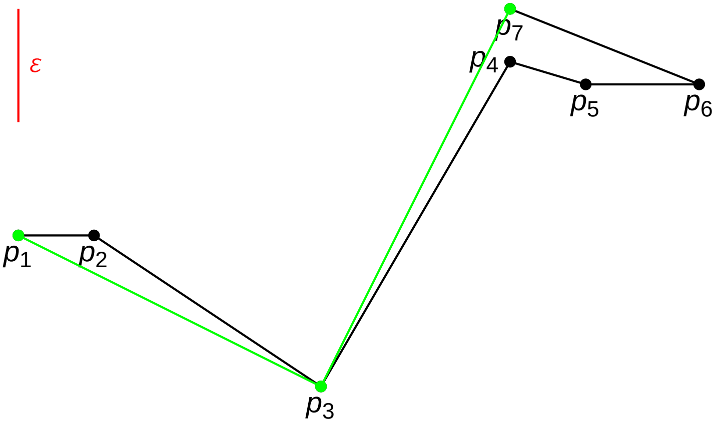

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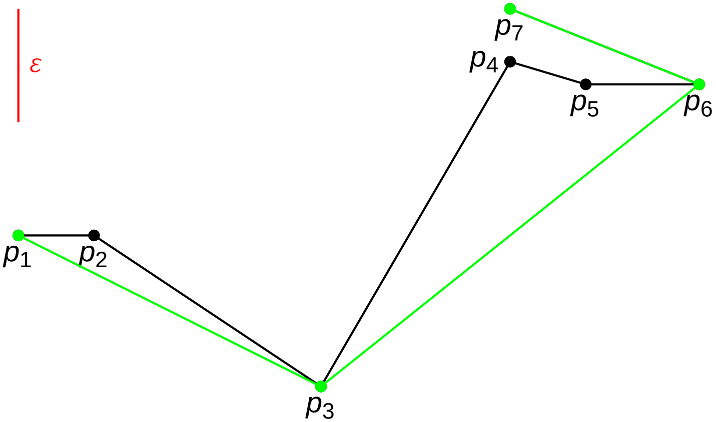

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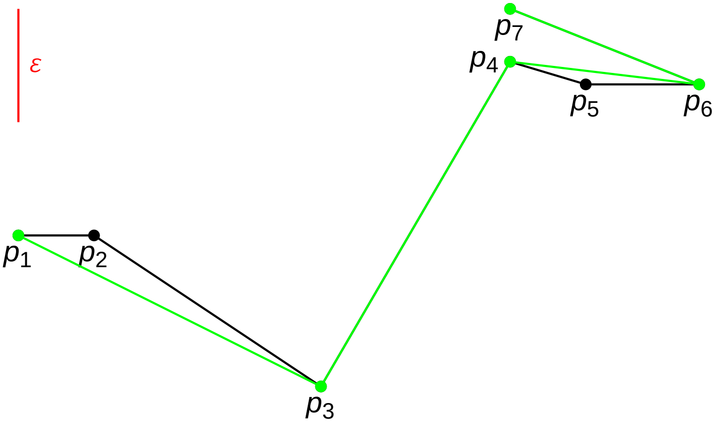
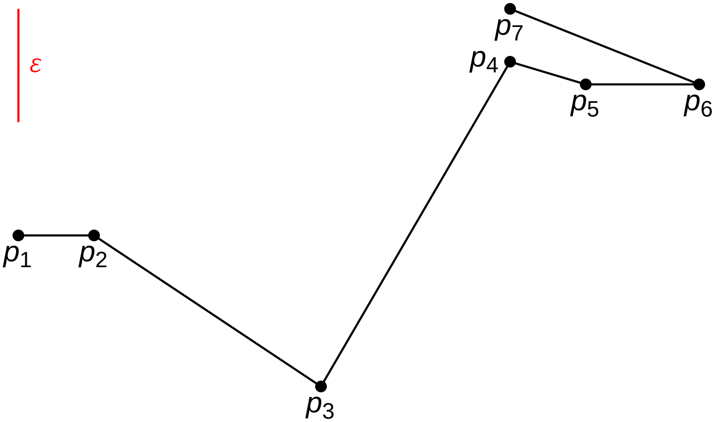


Imai-Iri



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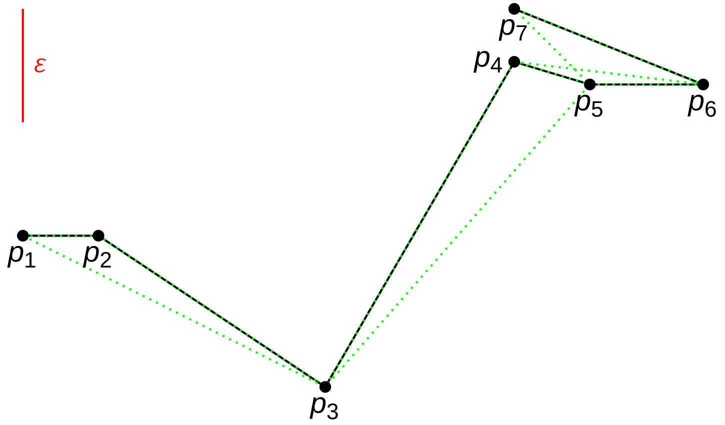

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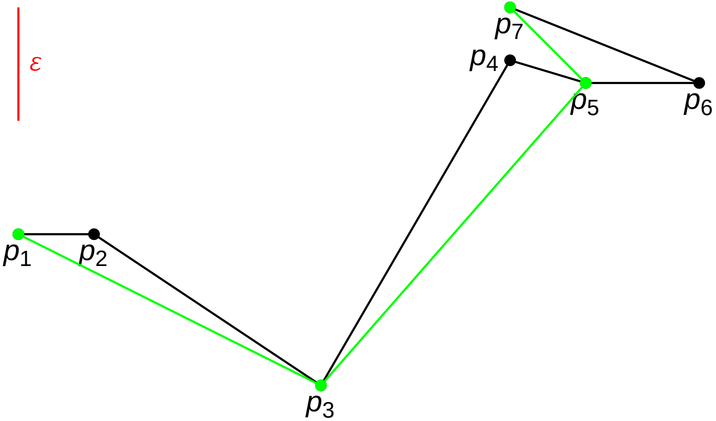

Approximation Quality

How well do these algorithms work?

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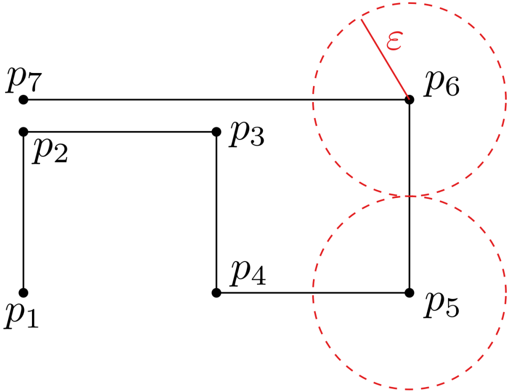
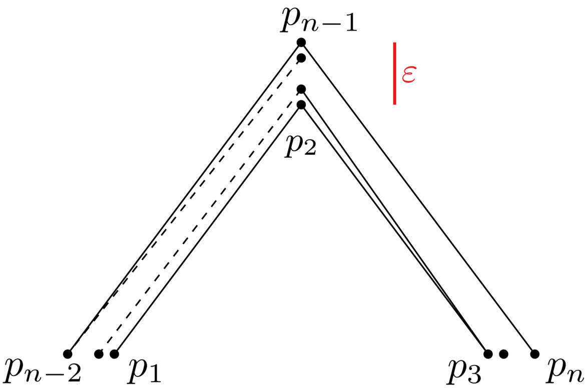


image by van Kreveld et al. [3]

Approximation Quality with Hausdorff Distance



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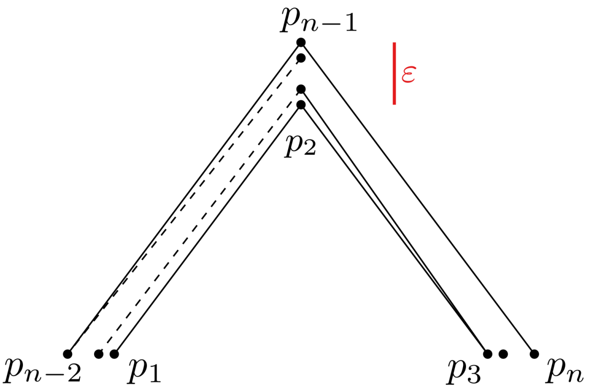

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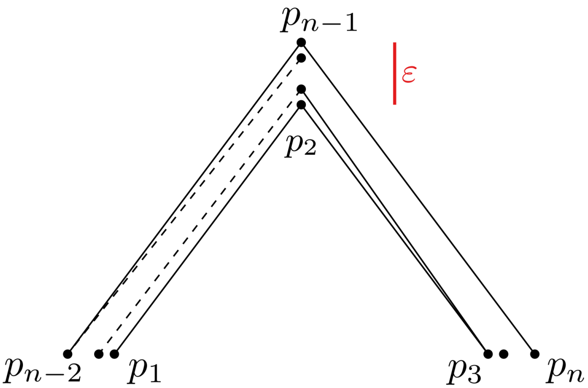
Theorem 1

there exists a polyline P with n vertices and $\varepsilon > 0$ such that $II_H(P, \varepsilon)$ has n vertices and $OPT_H(P, \varepsilon)$ has 3 vertices.

Approximation Quality with Hausdorff Distance



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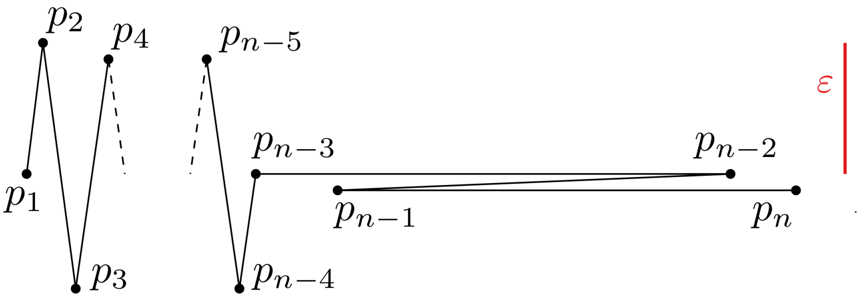
Theorem 1

For any $c > 1$, there exists a polyline P with n vertices and $\varepsilon > 0$ such that $II_H(P, c\varepsilon)$ has n vertices and $OPT_H(P, \varepsilon)$ has 3 vertices.

Approximation Quality with Fréchet Distance

Douglas-Peucker



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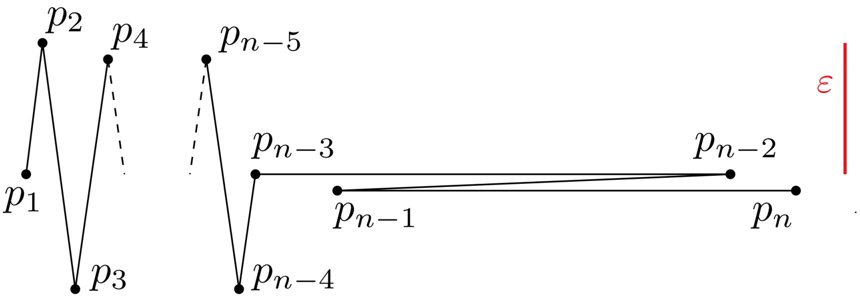

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Theorem 2

there exists a polyline P with n vertices and $\varepsilon > 0$ *such that* $DP_F(P, \varepsilon)$ *has n vertices and* $OPT_F(P, \varepsilon)$ *has 4 vertices.*

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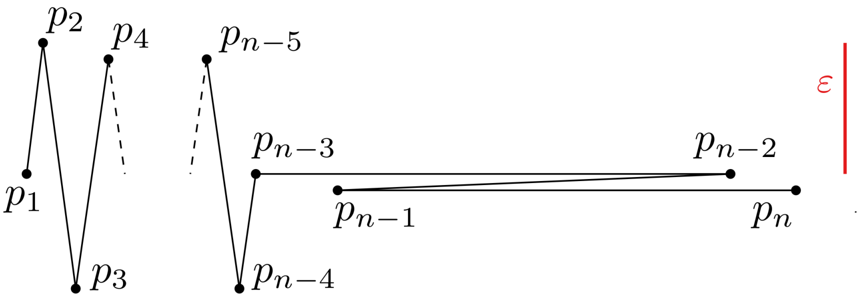

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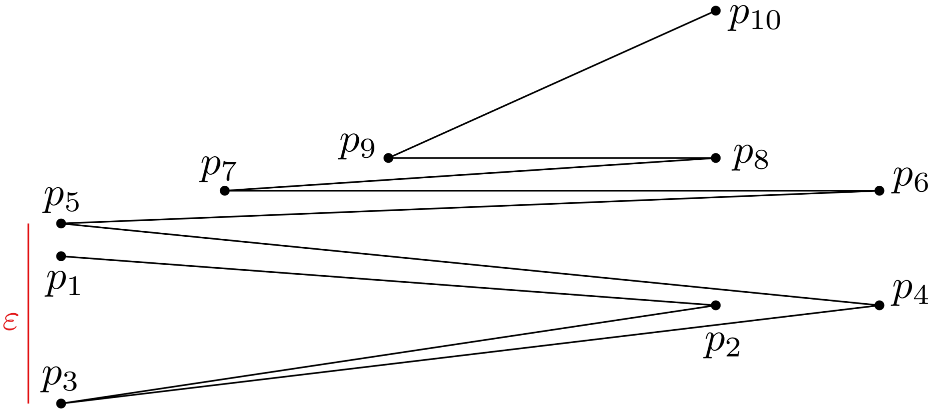

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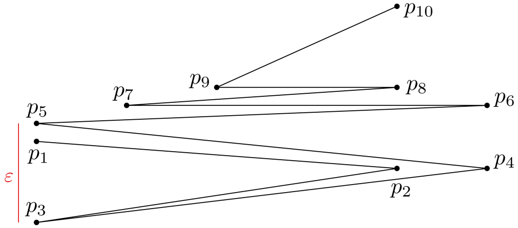

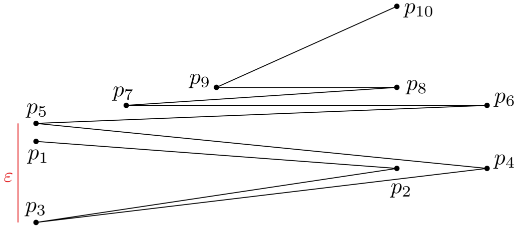

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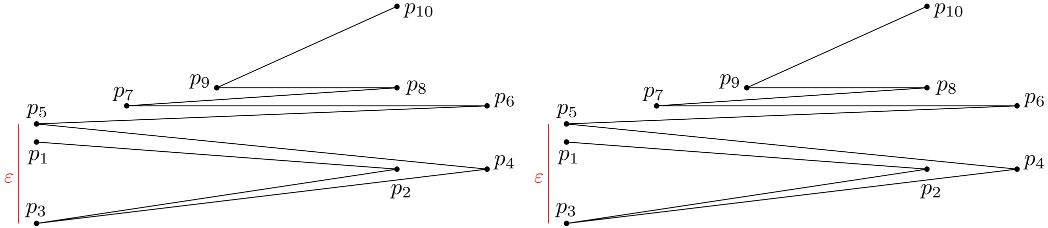

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Theorem 3

There exist $1 < c_1$, $1 < c_2$ *such that for any* $n > 0$*, a polyline* P *with* n *vertices and* $\varepsilon > 0$ *exist such that* $|II_F(P, c_1\varepsilon)| > c_2|OPT_F(P, \varepsilon)|$.

Approximation Quality with Fréchet Distance

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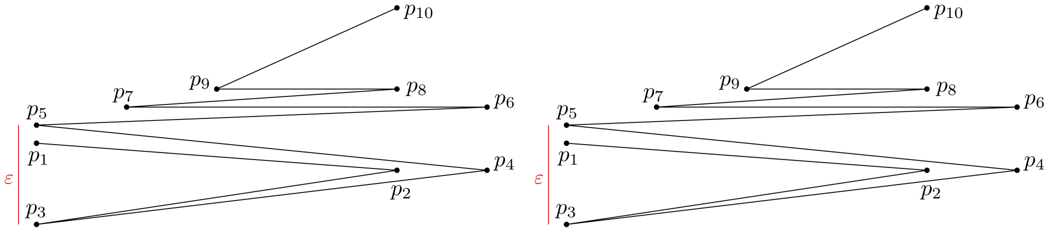

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Theorem 3

There exist $1 < c_1 < 4$, $1 < c_2$ such that for any $n > 0$, a polyline P with n vertices and $\varepsilon > 0$ exist such that $|II_F(P, c_1\varepsilon)| > c_2|OPT_F(P, \varepsilon)|$.

Algorithmic Complexity of the Optimal Solutions

- Computing the minimum length subsequence with Hausdorff distance at most ε is NP-hard.
- Computing the minimum length subsequence with Fréchet distance at most ε is possible in $\mathcal{O}(n^3)$ time [1].

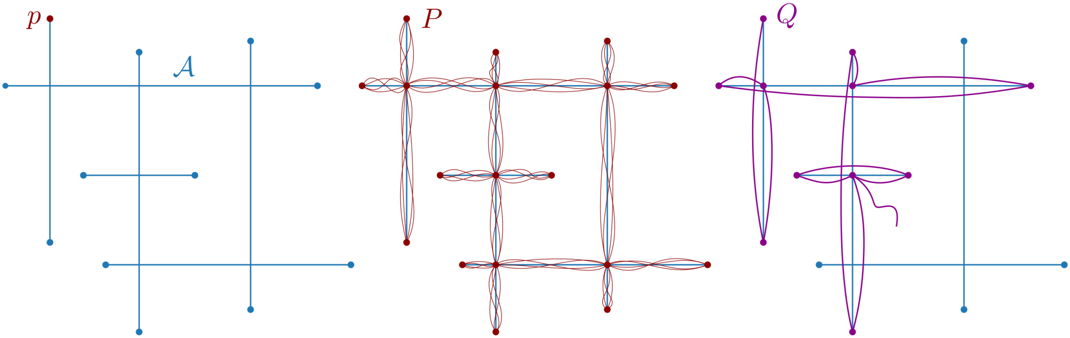
Thank you!

For Further Reading I

[1] Karl Bringmann and Bhaskar Ray Chaudhury. "Polyline Simplification has Cubic Complexity". In: *35th International Symposium on Computational Geometry (SoCG 2019)*. Ed. by Gill Barequet and Yusu Wang. Vol. 129. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2019, 18:1–18:16. ISBN: 978-3-95977-104-7. DOI: 10.4230/LIPIcs.SoCG.2019.18. URL: http://drops.dagstuhl.de/opus/volltexte/2019/10422.

[2] Marc van Kreveld, Maarten Löffler, and Lionov Wiratma. "On Optimal Polyline Simplification Using the Hausdorff and Fréchet Distance". In: *arXiv:1803.03550 [cs]* (Mar. 27, 2018). arXiv: 1803.03550. URL: http://arxiv.org/abs/1803.03550.

[3] Marc van Kreveld, Maarten Löffler, and Lionov Wiratma. "On Optimal Polyline Simplification Using the Hausdorff and Fréchet Distance". In: *JoCG* 11.1 (2020), pp. 1–25. URL: https://journals.carleton.ca/jocg/index.php/jocg/article/view/415.



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