The Spiroplot App, Casper van Dommelen, Marc van Kreveld, and Jérôme Urhausen

SoCG 2020 Multimedia Presentation, https://spiroplot.sites.uu.nl/

$P_1, P_2, \ldots \qquad P_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$

$$\begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \end{pmatrix}$$

$$P_1 \mapsto R_\varphi \left(P_1 - \frac{P_1 + P_1}{2} \right) + \frac{P_1 + P_2}{2}$$

$$R_\varphi = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

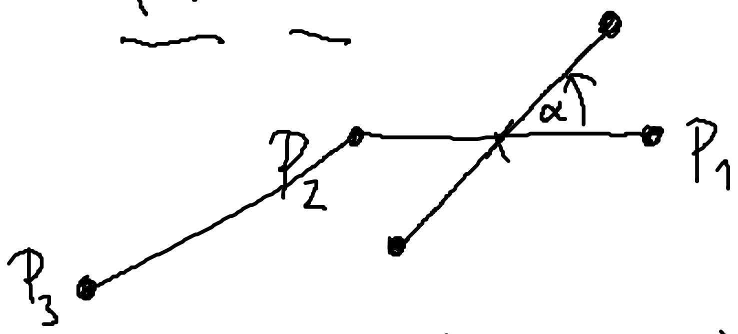
$P_2 \mapsto \ldots$

$$\begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{pmatrix} \mapsto \frac{1}{2} \begin{pmatrix} \cos\alpha + 1 & -\sin\alpha & -\cos\alpha + 1 & \sin\alpha \\ \sin\alpha & \cos\alpha + 1 & -\sin\alpha & -\cos\alpha + 1 \\ -\cos\alpha + 1 & \sin\alpha & \cos\alpha + 1 & -\sin\alpha \\ -\sin\alpha & -\cos\alpha + 1 & \sin\alpha & \cos\alpha + 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{pmatrix}$$

$$\alpha = 90^\circ: \qquad M = \frac{1}{2} \cdot \begin{pmatrix} 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cc} \frac{1}{2}\cos\alpha + \frac{1}{2} & -\frac{1}{2}\sin\alpha & -\frac{1}{2}\cos\alpha + \frac{1}{2} & \frac{1}{2}\sin\alpha & 0 & 0 \\ \frac{1}{2}\sin\alpha & \frac{1}{2}\cos\alpha + \frac{1}{2} & -\frac{1}{2}\sin\alpha & -\frac{1}{2}\cos\alpha + \frac{1}{2} & 0 & 0 \\ -\frac{1}{2}\cos\alpha + \frac{1}{2} & \frac{1}{2}\sin\alpha & \frac{1}{2}\cos\alpha + \frac{1}{2} & -\frac{1}{2}\sin\alpha & 0 & 0 \\ -\frac{1}{2}\sin\alpha & -\frac{1}{2}\cos\alpha + \frac{1}{2} & \frac{1}{2}\sin\alpha & \frac{1}{2}\cos\alpha + \frac{1}{2} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

$P_1 = P_2$ $\quad \underline{\begin{pmatrix}1\\0\\1\\0\end{pmatrix}}, \underline{\begin{pmatrix}0\\1\\0\\1\end{pmatrix}}$ fixpoints. $\quad \underline{\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}} = u \quad \frac{\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}}{\sqrt{2}} = v$



$u \mapsto \cos\alpha \cdot u + \sin\alpha \cdot v$

$v \mapsto -\sin\alpha \; u + \cos\alpha \cdot v$

$$M = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

in the basis $u, v, \begin{pmatrix}1\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\1\end{pmatrix}$

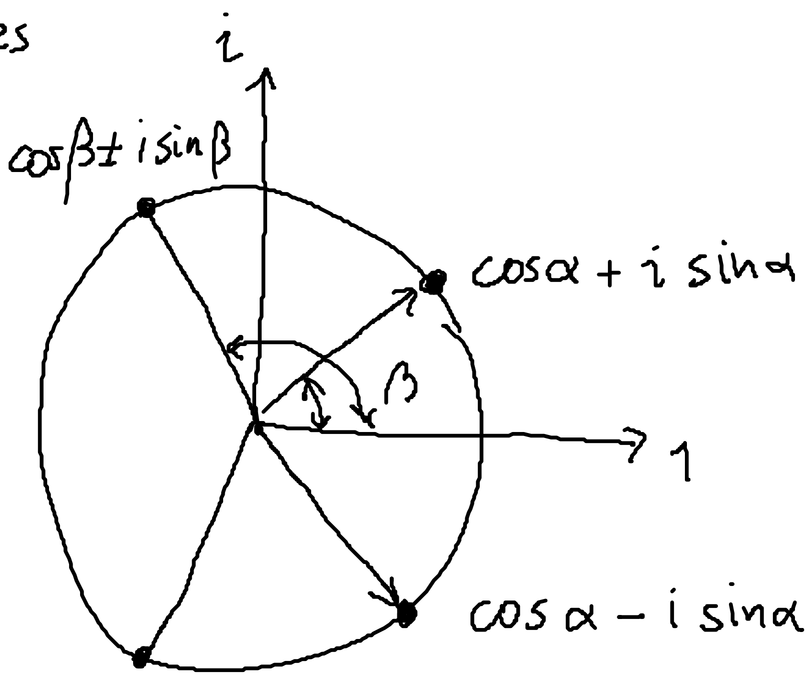
$n = 3 \qquad \mathbb{R}^6 \supseteq \mathbb{R}^4$

$\begin{pmatrix}1\\0\\1\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\1\\0\\1\end{pmatrix}$ ... fixpoints

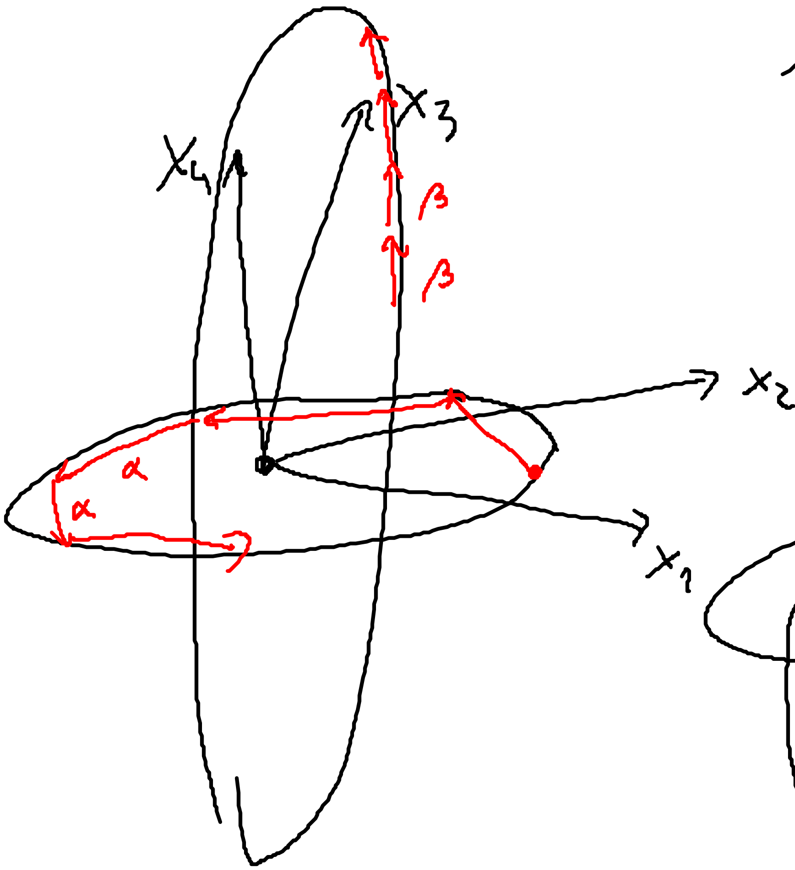
A general rotation matrix in high dimensions

$$R = \left(\begin{array}{cc|cc|cc|c}
\cos\alpha & -\sin\alpha & 0 & 0 & 0 & 0 & \cdots \\
\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 & \cdots \\
\hline
0 & 0 & \cos\beta & -\sin\beta & 0 & 0 & \cdots \\
0 & 0 & \sin\beta & \cos\beta & 0 & 0 & \cdots \\
\hline
0 & 0 & 0 & 0 & \cos\gamma & -\sin\gamma & \cdots \\
0 & 0 & 0 & 0 & \sin\gamma & \cos\gamma & \cdots \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)$$

eigenvalues

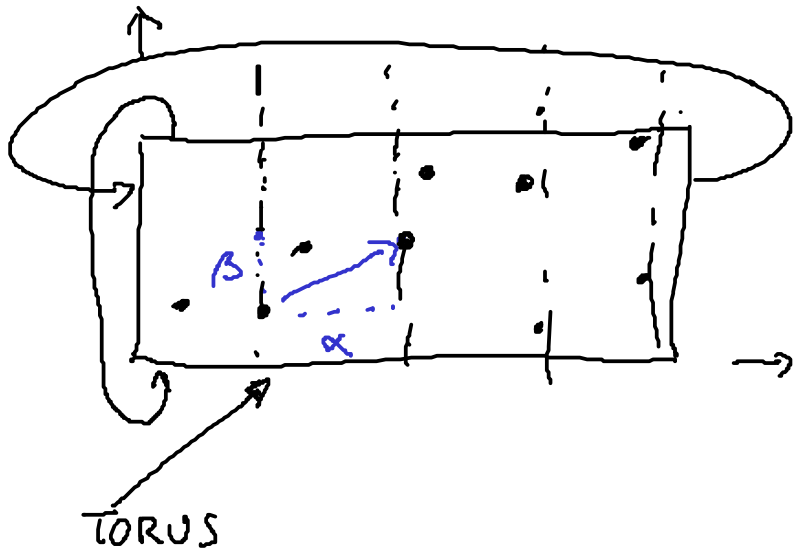


The orbit of a point $x^{(0)}$ under a rotation M in $\mathbb{R}^4$:



$$x^{(0)},\ M x^{(0)},\ M^2 x^{(0)},\ \ldots\ldots$$

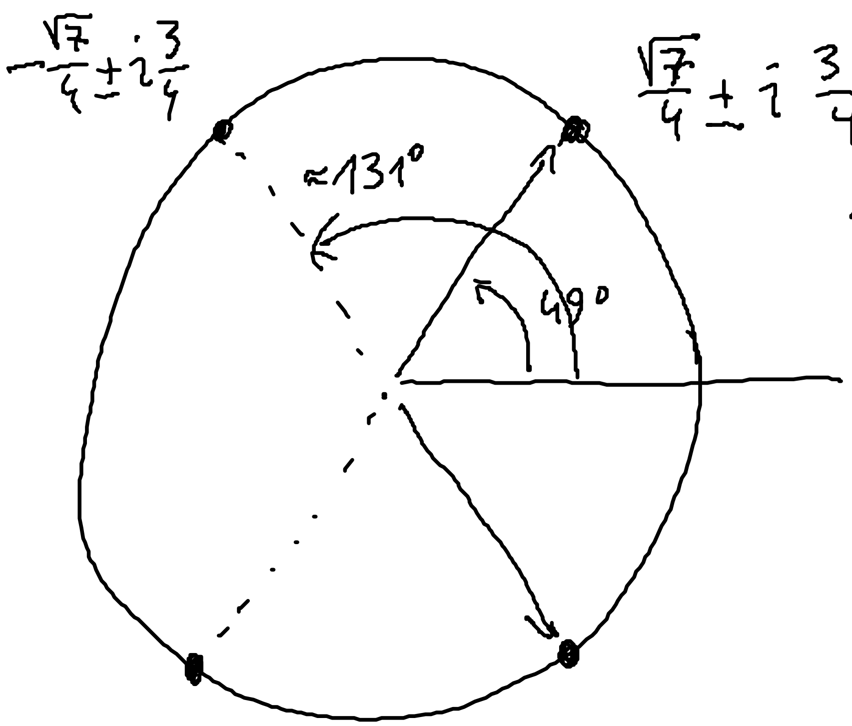
$$\begin{pmatrix} \left.\begin{matrix} x_1 \\ x_2 \end{matrix}\right] \to \alpha \\ \left.\begin{matrix} x_3 \\ x_4 \end{matrix}\right] \to \beta \end{pmatrix}$$



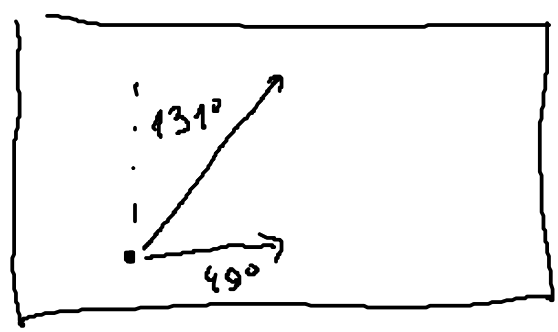
The case of 3 points, 2 rotations by 90°:

$$
\left(\begin{array}{cccc|cc}
1/2 & -1/2 & 1/2 & 1/2 & 0 & 0\\
1/2 & 1/2 & -1/2 & 1/2 & 0 & 0\\
1/2 & 1/2 & 1/2 & -1/2 & 0 & 0\\
-1/2 & 1/2 & 1/2 & 1/2 & 0 & 0\\
\hline
0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right)\cdot
\left(\begin{array}{cc|cccc}
1 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0\\
\hline
0 & 0 & 1/2 & -1/2 & 1/2 & 1/2\\
0 & 0 & 1/2 & 1/2 & -1/2 & 1/2\\
0 & 0 & 1/2 & 1/2 & 1/2 & -1/2\\
0 & 0 & -1/2 & 1/2 & 1/2 & 1/2
\end{array}\right)
$$

$$
= \frac{1}{2}\begin{pmatrix}
1 & -1 & 1 & 0 & 0 & 1\\
1 & 1 & 0 & 1 & -1 & 0\\
1 & 1 & 0 & -1 & 1 & 0\\
-1 & 1 & 1 & 0 & 0 & 1\\
0 & 0 & 1 & 1 & 1 & -1\\
0 & 0 & -1 & 1 & 1 & 1
\end{pmatrix}
$$



eigenvalues



2 × 49°

2 × (180 + 49°)