# Computergrafik



Rationale Splines (= Splines in homogenen Koordinaten)

$$\left.\begin{aligned} x(t) &= 2t^2+3t+4 \\ y(t) &= 5t^2+6t+7 \\ w(t) &= 8t^2+9t+1 \end{aligned}\right\}$$

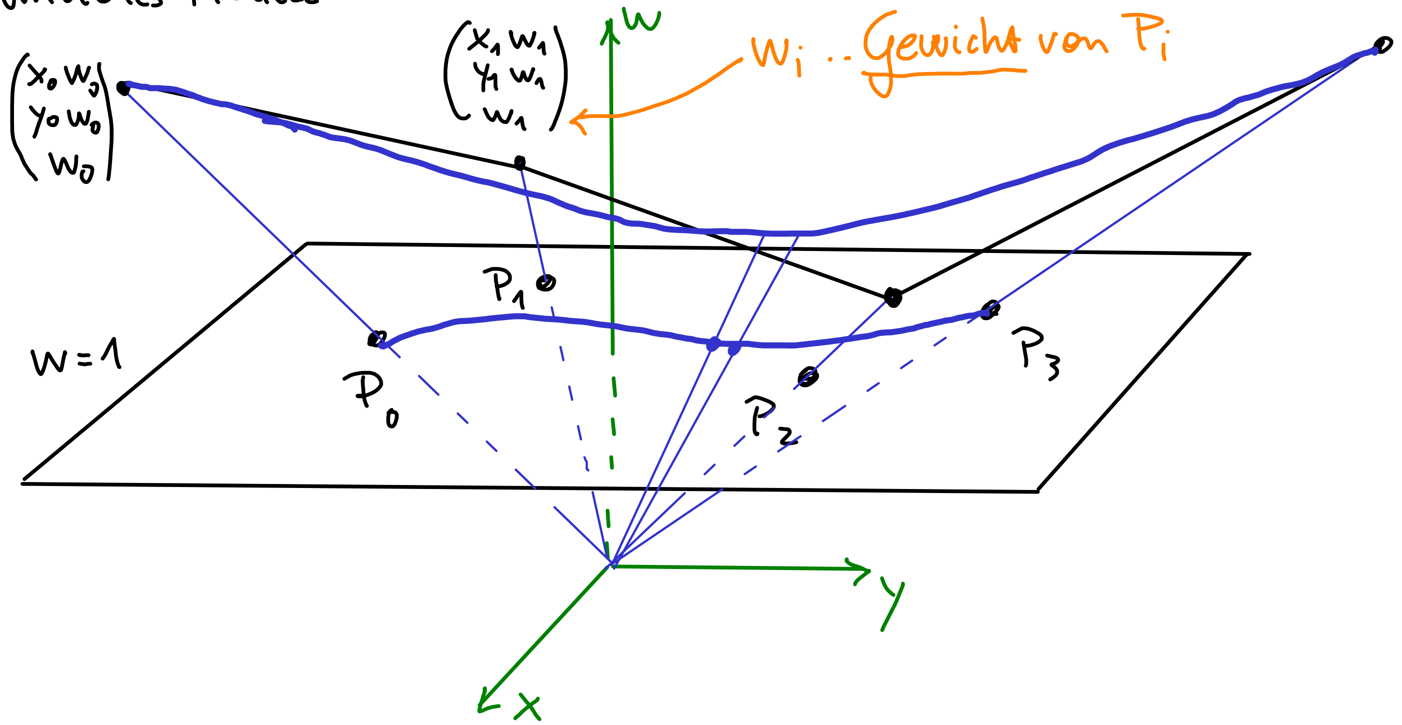
in kartesischen Koordinaten:

$$\begin{pmatrix} \dfrac{2t^2+3t+4}{8t^2+9t+1} \\[2ex] \dfrac{5t^2+6t+7}{8t^2+9t+1} \end{pmatrix}$$

rationale Funktionen von $t$

„nichtrationale Splines": (gewöhnliche) Polynomfunktionen

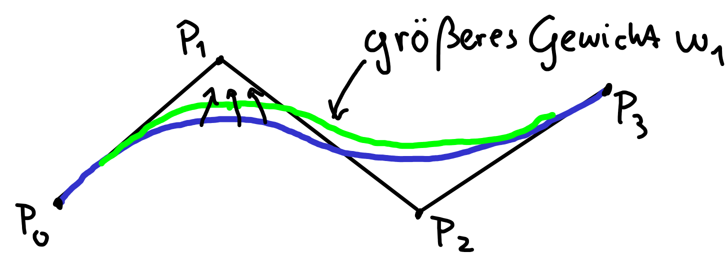
räumliches Modell




Eingabe: Folge von gewichteten Kontrollpunkten $(P_0, w_0), (P_1, w_1), \ldots (P_d, w_d)$

$w_i > 0$, $\qquad P_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$

Bézier-Spline für $\begin{pmatrix} x_i w_i \\ y_i w_i \\ w_i \end{pmatrix}$



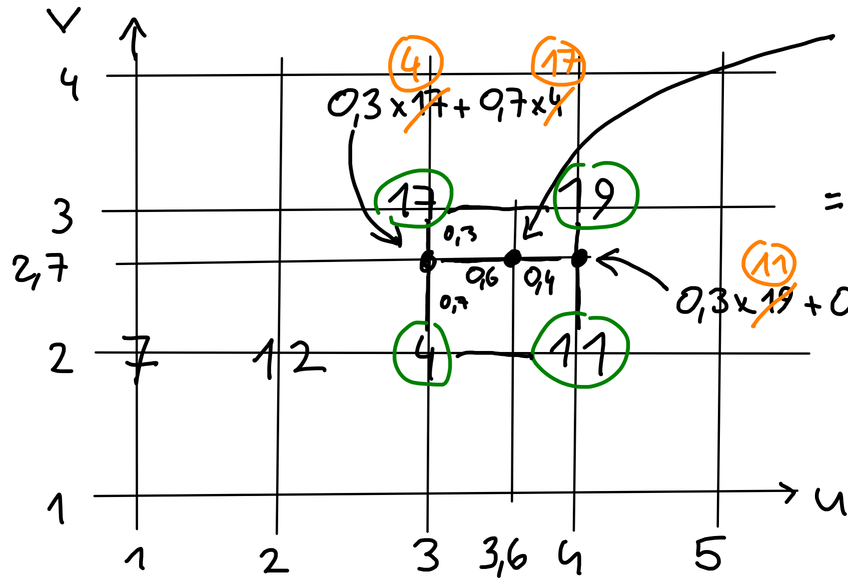
## Splineflächen

$[0,1]^2 \to \mathbb{R}^3 \qquad \begin{pmatrix} \boxed{x(u,v)} \\ y(u,v) \\ z(u,v) \end{pmatrix} \leftarrow \quad [0,1]^2 \to \mathbb{R}$

NACHTRÄGLICH KORRIGIERT

in Orange

Spezialfall: Bilineare Interpolation




$\overset{0{,}4}{\cancel{0{,}6}} \times \left(0{,}3 \times \overset{4}{\cancel{19}} + 0{,}7 \times \overset{17}{\cancel{4}}\right)$

$+\, \overset{0{,}6}{\cancel{0{,}4}} \times \left(0{,}3 \times \overset{11}{\cancel{17}} + 0{,}7 \times \overset{19}{\cancel{11}}\right)$

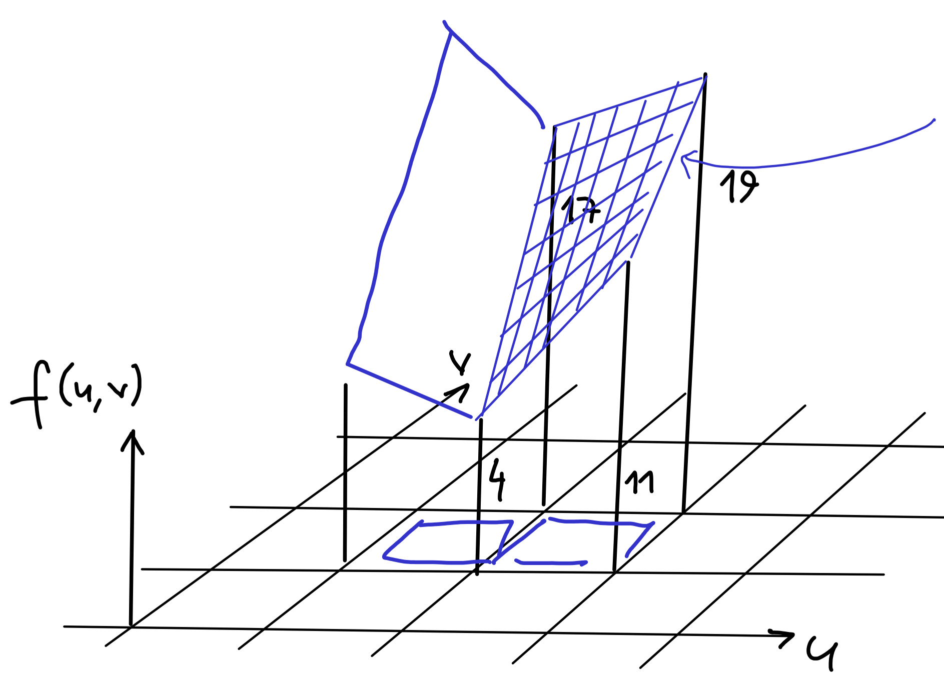
$= 0{,}18 \times \underset{11}{\cancel{19}} + 0{,}42 \times \underset{19}{\cancel{4}} + 0{,}12 \times \underset{4}{\cancel{17}} + 0{,}28 \times \underset{17}{\cancel{11}}$

> Anwendung:
> Interpolation einer Textur

$$f(i+x, j+y) = (1-x)(1-y)\cdot f(i,j) + (1-x)y\cdot f(i,j+1) + x(1-y)\cdot f(i+1,j) + xy\cdot f(i+1,j+1)$$

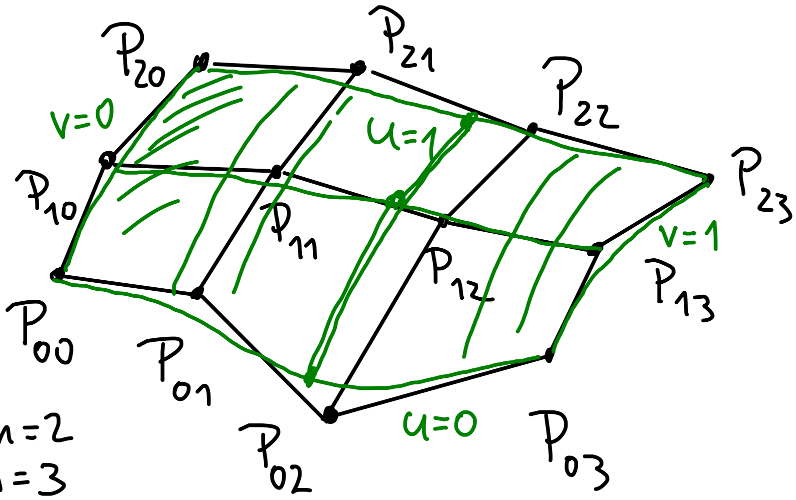
> $0 \le x,y \le 1$
> $i,j \in \mathbb{Z}$

$$g(i+x) = (1-x)\cdot g(i) + x\cdot g(i+1)$$




hyperbolisches Paraboloid

Bézier-Splineflächen




$m=2$
$n=3$

Eingabe:
Ein Gitter von Kontrollpunkten
$P_{ij}\,,\ 0 \le i \le m\,,\ 0 \le j \le n$

Spline-Kurven:

$$\boxed{X(t) = \sum_{i=0}^{d} B_i^d(t)\, P_i}$$

$$\boxed{X(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} B_i^m(u) \cdot B_j^n(v)\, P_{ij}} \qquad 0 \le u,v \le 1$$

$$\sum_{i=0}^{m} \sum_{j=0}^{n} B_i^m(u) \cdot B_j^n(v) = \underbrace{\sum_{i=0}^{m} B_i^m(u)}_{1} \cdot \underbrace{\sum_{j=0}^{n} B_j^n(v)}_{1} = 1$$

$$X(u,v) = \sum_{i=0}^{m} B_i^m(u) \cdot \underbrace{\left[\sum_{j=0}^{n} B_j^n(v)\, P_{ij}\right]}_{Q_i(v)}$$

= Spline-Kurve mit Kontrollpolygon $Q_0(v)\, Q_1(v) \ldots Q_m(v)$

$Q_i(v)$ wandert auf Spline-Kurve mit Kontrollpolygon $P_{i0}\, P_{i1} \ldots P_{in}$.