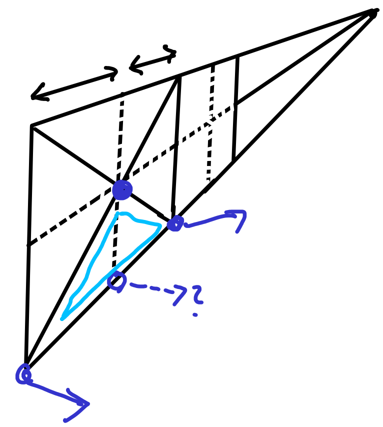
Lineare Interpolation in Weltkoordinaten



$$I: \mathbb{R}^3 \to \mathbb{R}$$

$I(x,y,z)$ affin-linear

$$I(x,y,z) = ax + by + cz + d$$

$$\hat{I}: \mathbb{R}^4 \to \mathbb{R}$$

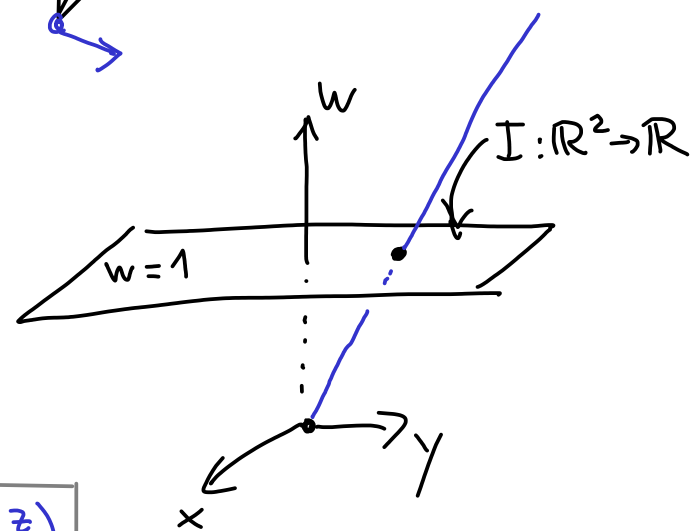
$\hat{I}(x,y,z,w)$ linear

$$\hat{I}(x,y,z,w) = ax + by + cz + dw$$

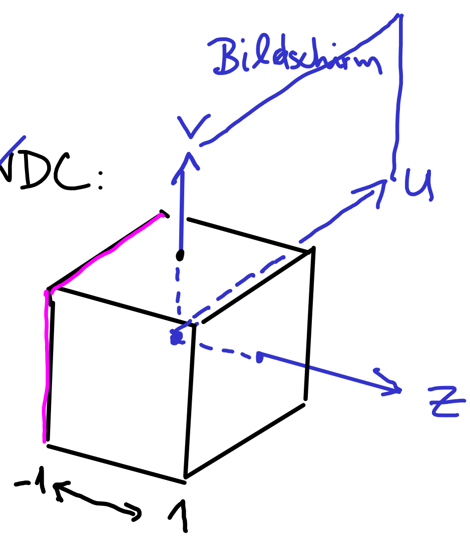
$$\hat{I}(x,y,z,1) = I(x,y,z)$$

$$\hat{I}(\lambda x, \lambda y, \lambda z, \lambda w) = \lambda \hat{I}(x,y,z,w)$$

$$\boxed{\hat{I}(x,y,z,w) = w \cdot \hat{I}\left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}, 1\right) = w \cdot I\left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)}$$



~~NDC~~:



$$M \cdot \begin{pmatrix} x \\ y \\ z \\ w=1 \end{pmatrix} = \begin{pmatrix} u^0 \\ v^0 \\ z^0 \\ w^0 \end{pmatrix} \hookrightarrow \begin{pmatrix} \boxed{\begin{matrix} u^0/w^0 \\ v^0/w^0 \end{matrix}} \\ \cancel{z^0/w^0} \\ 1 \end{pmatrix} \begin{matrix} = u \\ = v \\ = \cancel{z} \\ \end{matrix}$$

$$M^{-1} \begin{pmatrix} u \\ v \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \cdot \frac{1}{w_0}$$

$$\boxed{w' = \frac{1}{w_0}}$$

$$\hat{I}\left(M^{-1} \begin{pmatrix} u \\ v \\ z \\ 1 \end{pmatrix}\right) \ldots$$ affin-lineare Funktion in u, v

$w' = 4.$ Komponente von $M^{-1} \begin{pmatrix} u \\ v \\ z \\ 1 \end{pmatrix} \ldots$

Interpolationsalgorithmus:

Dreieck ABC, mit I^R_A, I^R_B, I^R_C

Beim Umwandeln der Punkte A, B, C von Weltkoordinaten in Bildschirmkoordinaten mit der Formel $M \begin{pmatrix} x_A \\ y_A \\ z_A \\ 1 \end{pmatrix} = \begin{pmatrix} \vdots \\ w_A^0 \end{pmatrix}$ ergaben sich Werte w_A^0, w_B^0, w_C^0.

$$w'_A = \frac{1}{w_A^0} \qquad w'_B = \frac{1}{w_B^0} \qquad w'_C = \frac{1}{w_C^0}$$

$$\hat{I}^R_A = I^R_A \cdot w'_A \qquad \hat{I}^R_B = I^R_B \cdot w'_B \qquad \hat{I}^R_C = I^R_C \cdot w'_C$$

Interpoliere $\hat{I}^R$, w' in Bildschirmkoordinaten

Für jedes Pixel $I^R := \frac{\hat{I}^R}{w'}$.

analog für I^G, I^B etc.

$$I(u,v) = \frac{au + bv + c}{du + ev + f}$$, gebrochen lineare Funktion