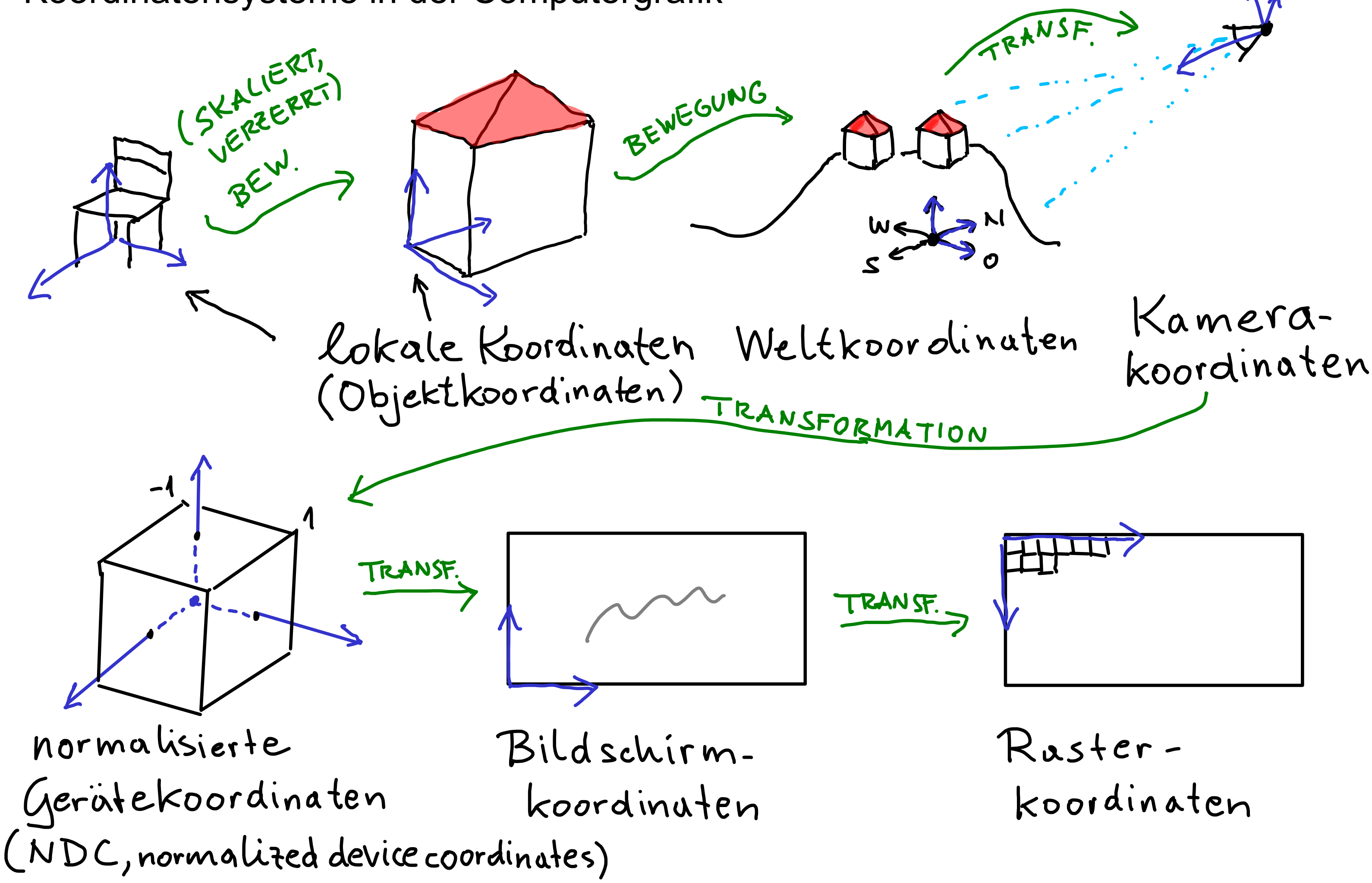
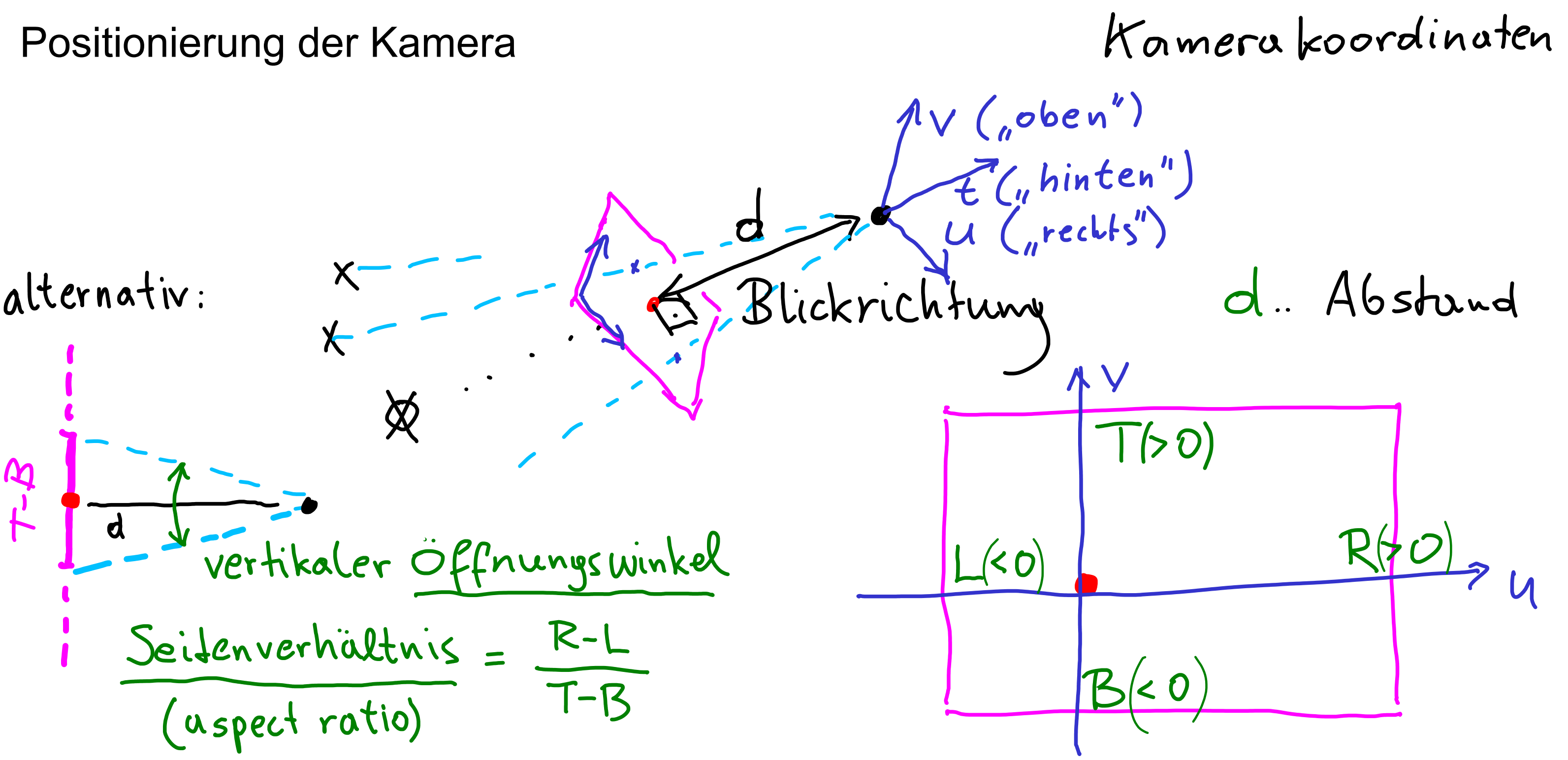
Computergrafik

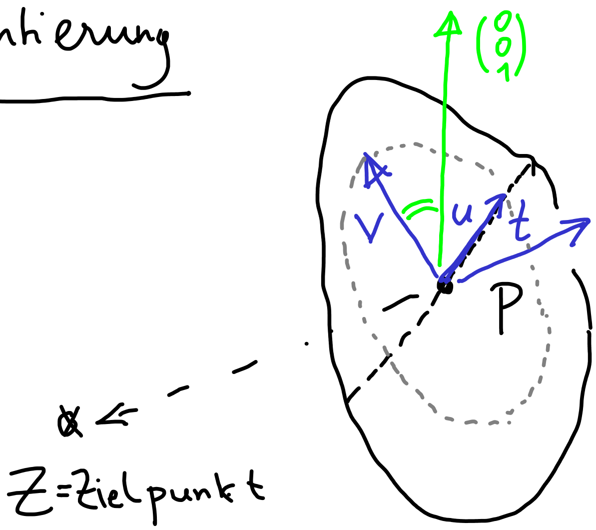
Koordinatensysteme in der Computergrafik



Positionierung der Kamera



natürliche Kameraorientierung

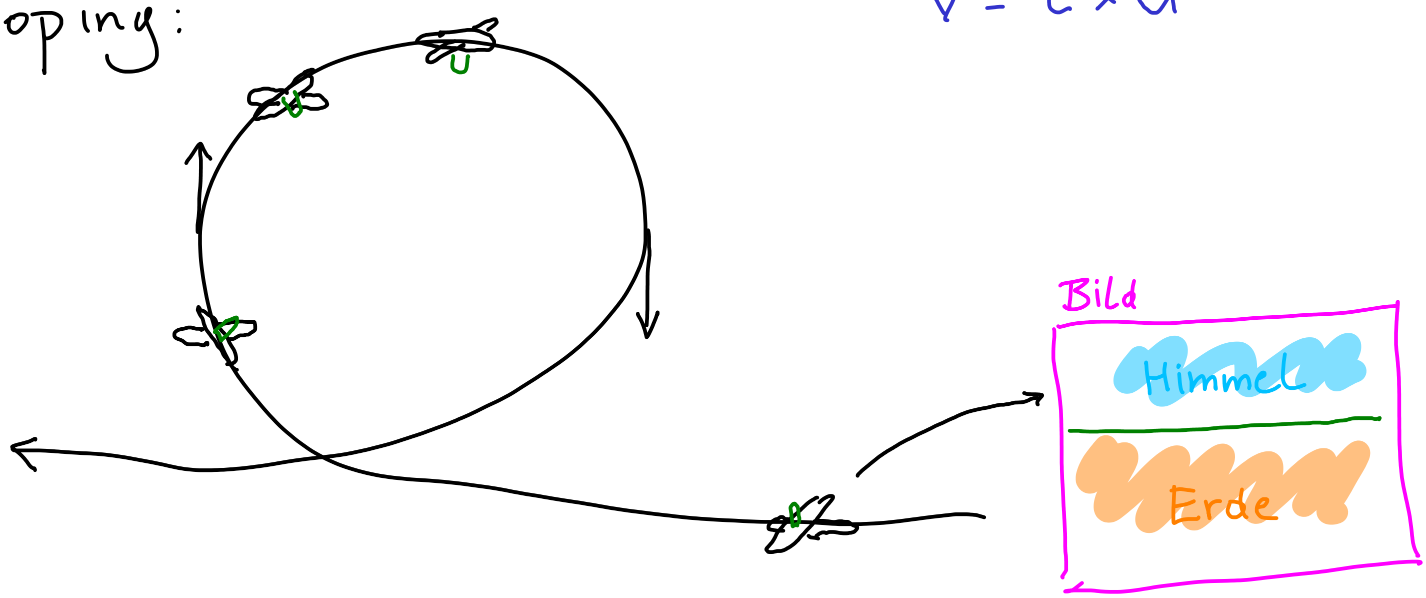


$$t = \frac{P - Z}{\|P - Z\|}$$

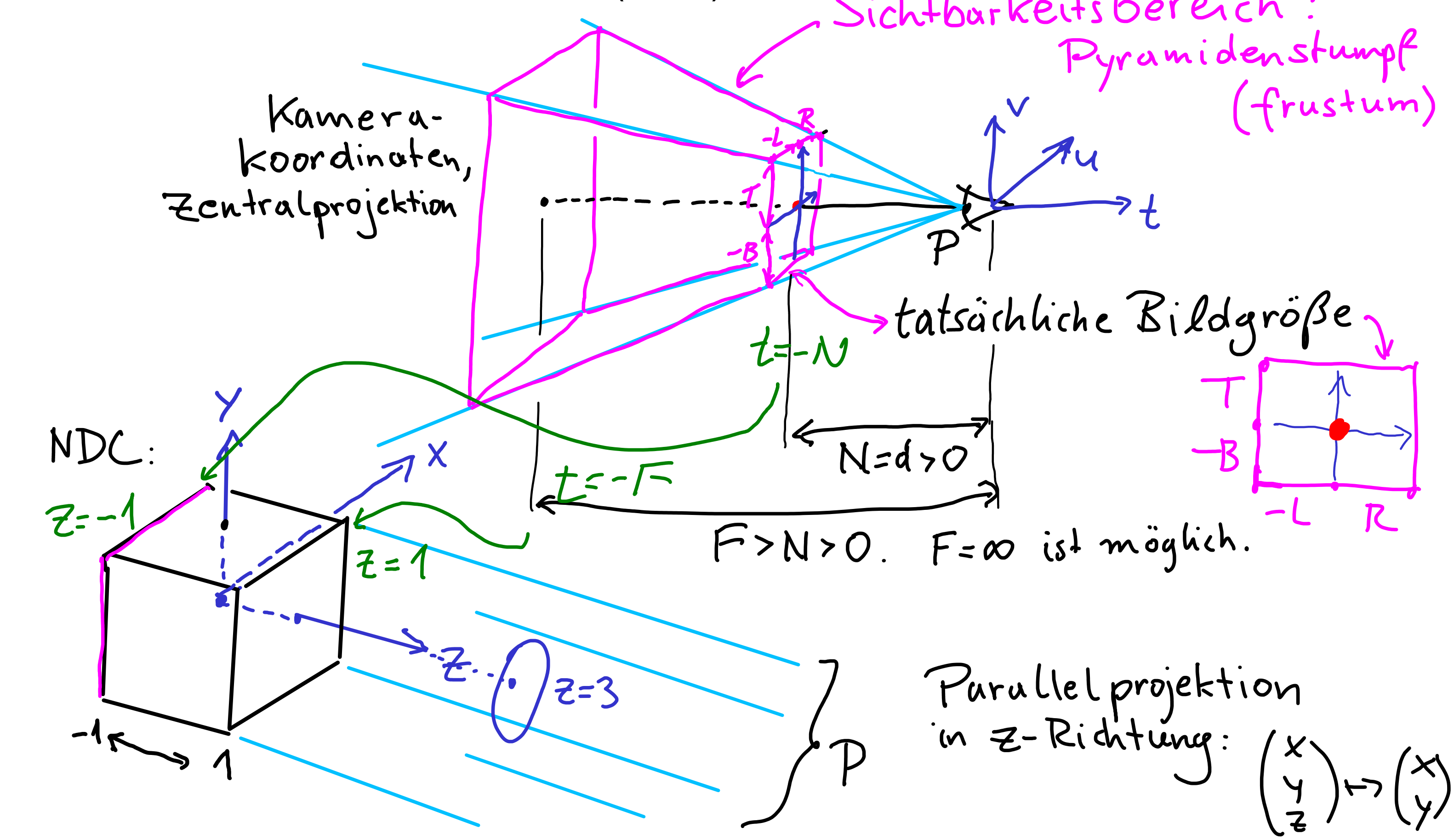
$$u = \frac{\begin{pmatrix}0\\0\\1\end{pmatrix} \times t}{\left\| \begin{pmatrix}0\\0\\1\end{pmatrix} \times t \right\|}$$

$$v = t \times u$$

Looping:



Normalisierte Gerätekoordinaten (NDC)



VEREINFACHUNG: $B=L=-1,\ T=R=1,\ N=1.$

$$\begin{pmatrix}x\\y\\z\\w\end{pmatrix} = \underbrace{\begin{pmatrix} c&0&0&0\\0&d&0&0\\0&0&a&b\\0&0&-1&0\end{pmatrix}}_{M}\begin{pmatrix}u\\v\\t\\w_0\end{pmatrix}$$

$$P=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\mapsto\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\cdot\lambda$$

Fernpunkte in u- und v-Richtung bleiben erhalten:

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\mapsto\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\cdot\lambda_1,\quad \begin{pmatrix}0\\1\\0\\0\end{pmatrix}\mapsto\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\cdot\lambda_2$$

$$\begin{pmatrix}0\\0\\-F\\1\end{pmatrix}\mapsto\begin{pmatrix}0\\0\\1\\1\end{pmatrix}\cdot\lambda_4=\begin{pmatrix}0\\0\\-Fa+b\\F\end{pmatrix}\qquad \lambda_4=F,\quad F=-Fa+b$$

$$\begin{pmatrix}0\\0\\-1\\1\end{pmatrix}\mapsto\begin{pmatrix}0\\0\\-1\\1\end{pmatrix}\cdot\lambda_3=\begin{pmatrix}0\\0\\b-a\\1\end{pmatrix}\qquad \lambda_3=1$$

$$b-a=-1$$

$$\boxed{a=-\frac{F+1}{F-1}\qquad b=-\frac{2F}{F-1}}$$

$$\begin{pmatrix}1\\1\\-1\\1\end{pmatrix}\mapsto\begin{pmatrix}1\\1\\-1\\1\end{pmatrix}\cdot\lambda_5=\begin{pmatrix}c\\d\\-a+b\\1\end{pmatrix}\qquad \lambda_5=1\qquad \boxed{c=d=1}$$

Transformationsmatrix von Kamerakoordinaten in NDC

$$M = \begin{pmatrix} \frac{2N}{R-L} & 0 & \frac{R+L}{R-L} & 0 \\ 0 & \frac{2N}{T-B} & \frac{T+B}{T-B} & 0 \\ 0 & 0 & -\frac{F+N}{F-N} & -\frac{2FN}{F-N} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

nachträgliche Korrektur

$$M^{-1} = \begin{pmatrix} \frac{R-L}{2N} & 0 & 0 & \frac{R+L}{2N} \\ 0 & \frac{T-B}{2N} & 0 & \frac{T+B}{2N} \\ 0 & 0 & 0 & -1 \\ 0 & 0 & \frac{1}{2F}-\frac{1}{2N} & \frac{1}{2F}+\frac{1}{2N} \end{pmatrix}$$

$F = \infty$ ist erlaubt.

Bilder der Fernpunkte.

$$\begin{pmatrix} u \\ v \\ t \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} \frac{u\,2N + t\,(R+L)}{R-L} \\ \frac{v\,2N + t\,(T+B)}{T-B} \\ -t\,\frac{F+N}{F-N} \\ -t \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ w_0 \end{pmatrix} \hat{=} \begin{pmatrix} * \\ * \\ \frac{F+N}{F-N} \\ 1 \end{pmatrix}$$

Bsp. $N=1 \quad F=2$

$\Rightarrow z = 3$