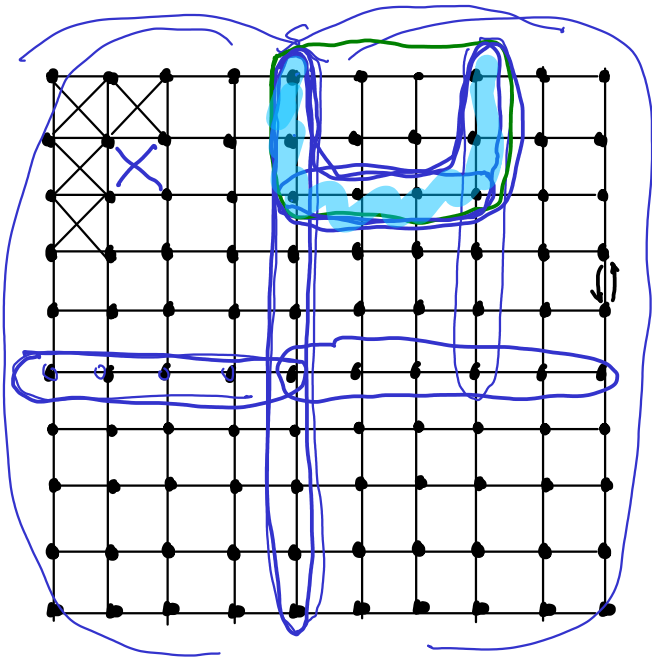


Generalized nested dissection

Lipton, Rose, Tarjan 1979, Nested dissection: Alan George 1973



$$x_i = \frac{1}{4} (\sum_{i \sim j} 4\text{-neighbors})$$

→ Tutte embeddings

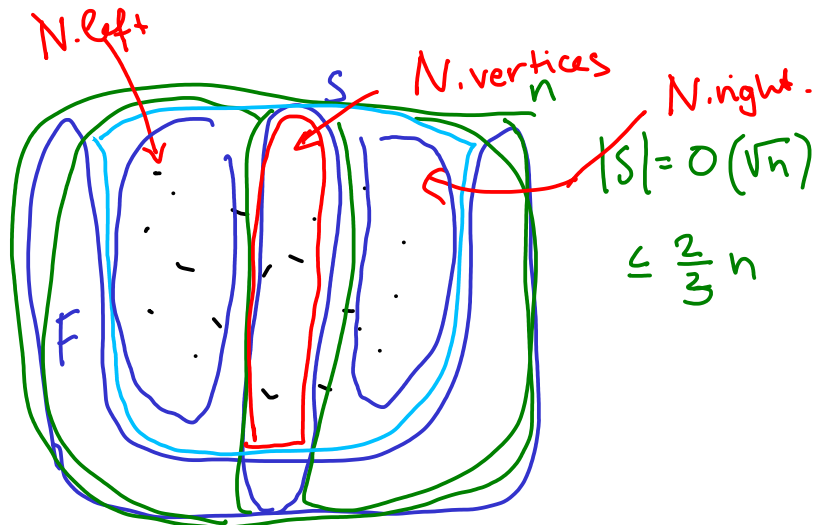
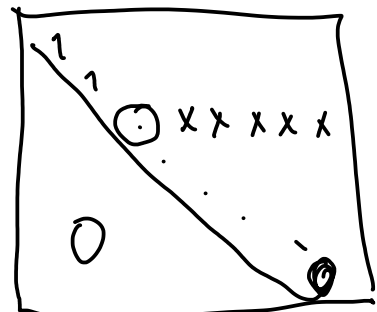
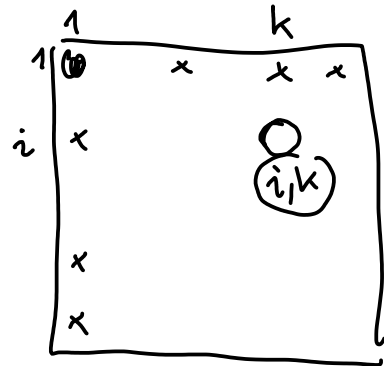
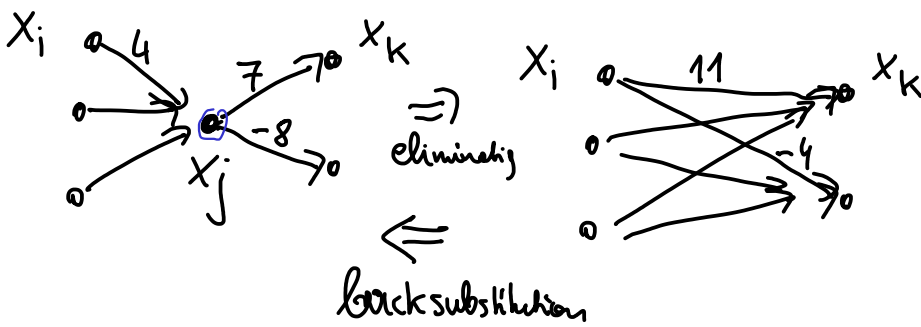
$$x_j \leq \sqrt{a_{ij}} x_i + c_{ij}$$



$$x_1 \leq x_1 + 5$$

$O(\sqrt{n})$ -separators + n^w ... $n \times n$ matrix multiplication } $\Rightarrow O(n^{w/2})$?

TODAY: $O(\sqrt{n})$ -separators $\Rightarrow O(n^{3/2})$ time, $O(n)$ space.



elim_all (N)

elim_all (N.left)

elim (N.right)

$\forall v \in N.vertices$

eliminate (v)

bs_all (N)

$\forall v \in N.vertices$

backsub (v)

bs_all (N.left)

--- (N.right)

$T(n, f)$

 $\underbrace{\hspace{2em}}_{\text{Time}} \quad s = \sqrt{n}$

 Space

 $s (s+f)^2$

 s^2

... 2 subproblems $T(n_1, f_1)$

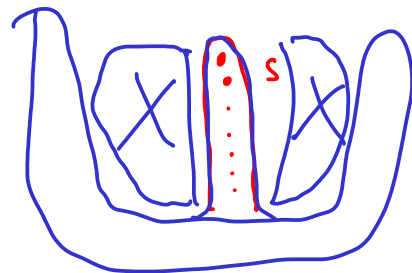
$T(n_2, f_2)$

$$n_1 + n_2 \leq n + O(\sqrt{n})$$

$$f_1 + f_2 \leq f + 2 \cdot O(\sqrt{n})$$

$$S(n) = (\sqrt{n})^2 + \cancel{2}^1 S\left(\frac{n}{2}\right) = n \log n$$

$$T(n) = (\sqrt{n})^3 + 2T\left(\frac{n}{2}\right) = n^{3/2}$$



$n^{2/3}$

$S(n^{4/3})$

$\text{elim_all}^D(N)$

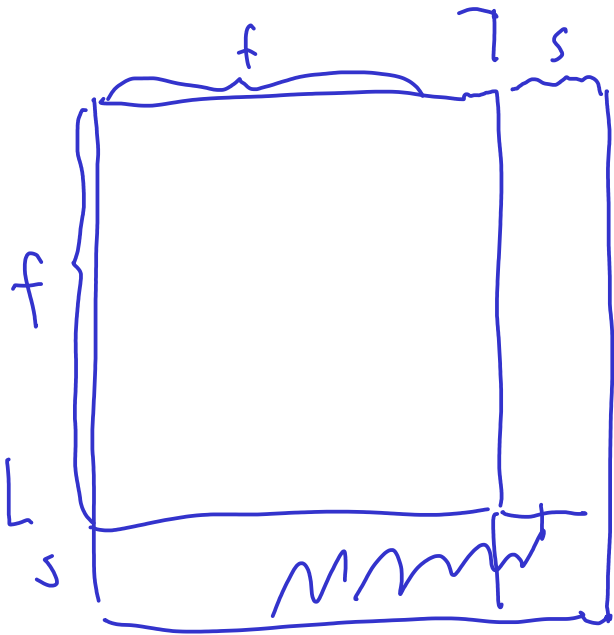
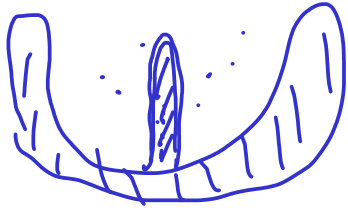
$\text{elim_all}^D(N.\text{left})$

$\text{elim}^D(N.\text{right})$

$\forall v \in N.\text{vertices}$

$\left| \begin{array}{l} \text{eliminate}(v) \\ \text{forget}(v) \end{array} \right.$

delete all inserted edges incident to v .



$\text{bs_all}(N)$

$\forall v \in N.\text{vertices}$

$\leftarrow \text{backsub}(v)$

$\text{bs_all}(N.\text{left})$

$\text{---} (N.\text{right})$

$e+Bs(N)$

$\text{elim_all}^D(N.\text{left})$

$\text{elim_all}^D(N.\text{right})$

$\forall v \in N.\text{vertices}$

$\left| \begin{array}{l} \text{eliminate}(v) \\ \leftarrow \end{array} \right.$

$\forall v \in N.\text{vertices}$

$\text{backsub}(v)$

$e+Bs(N.\text{left})$

$e+Bs(N.\text{right})$

$\forall v \in N.\text{vertices} :$

$\text{forget}(v)$