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Making Change in 2048

Our assumptions:

- Set A of allowable tile values, $1 \in A$
- And 1 is spawned
- n indistinguishable cells
  - ↳ no relation to each othr other in any way
  - ↳ can contain a value out of A
  - ↳ initially empty

Moves:

1. We place 1 into an empty cell
2. Merge any sets of cells, whose sum is in A

$\Rightarrow AGG(n, A)$

superset of 2048

at each step: The total value increases by one

Eager sequency: Do each merge at first step possible

-> all sequences can be made eager

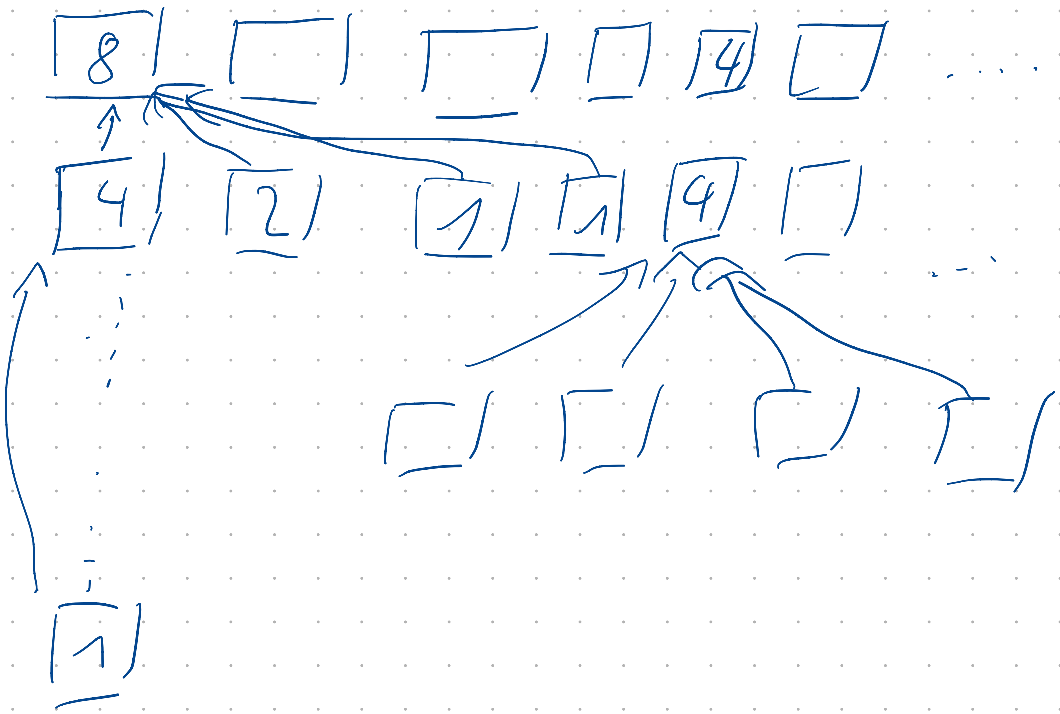
=> At step we do only a single merge, this merge always uses the new 1

merge($\boxed{x}$ $\boxed{y}$ $\boxed{z}$ $\boxed{1}$)

$\boxed{}$ $\boxed{x+y+z+1}$ $\boxed{}$ $\boxed{}$

single - tile - first - strategy.

Assume some position of AGG(n, A), which is reachable

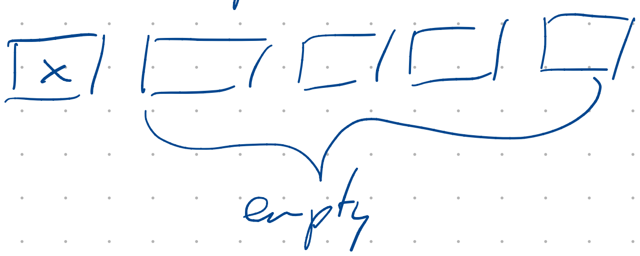


- We can reorder steps to construct tile by tile

first tile in AGG(n, A)

remaining AGG(n-1, A)

Let P_x be a position:



$A = \mathbb{N}$

$x = 7$

$y = 6$

$7 = 3 + 4 \qquad 7 = 6 + 1$

Lemma 1: Let $A = \{1, \dots y, x, \dots\}$

P_x is reachable in $AGG(n, A)$ $\Leftrightarrow$ (1) P_y is reachable in $AGG(n, A)$

(2) A position of value $x - y$ is reachable in $AGG(n-1, A)$

Proof:

$\Leftarrow$

$\Rightarrow$ (1) If we reach P_x, then we had the total value of y at some point. $\Rightarrow$ merge

(2) Before we reach P_x, we have a total value of $x-1$ and one empty field

We can decompose $x-1$ into

P_z, constructable in $AGG(n, A)$

$x-1$: $\boxed{z}\ \underbrace{\square\ \square\ \square\ \square\ \square}_{x-1-z}\ \overset{\text{empty}}{\square}$

$\}$ constructable in $AGG(n-1, A)$

case 1: $z = y$: Fill 1 into empty field: $x-1-y+1 = x-y$

case 2: $z < y$: $x-1-z > x-y$

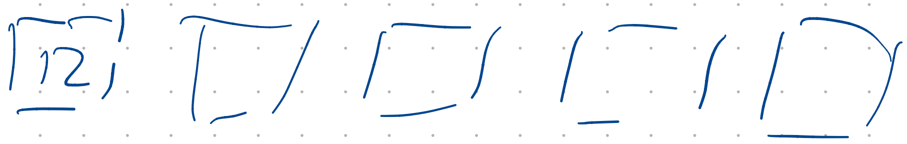
Corollary: We can construct P_x via P_y

$\text{Single}(n, A)$: the largest x in P_x possible in $\text{AGG}(n, A)$

$$\text{Total}(n, A) = \text{Single}(n, A) + \text{Single}(n-1, A) + \dots$$

$$= \sum_{i=1}^{n} \text{Single}(n, A)$$

if the gap in A doesn't increase over n we never terminate



We can always start with the largest tile in single tile first

$v_n, \dots, v_1$

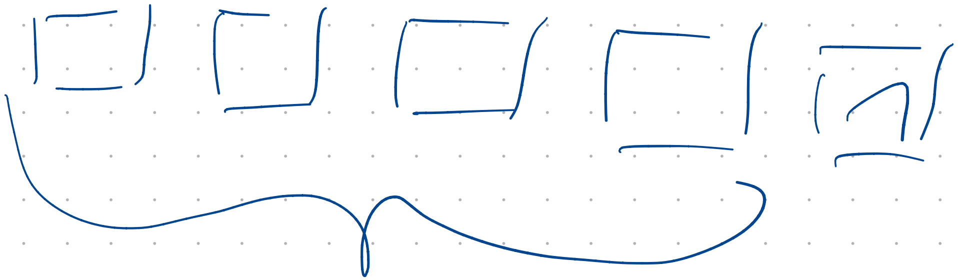
$v_i < v_j$ with $i > j$

$\text{Single}(0, A) = \text{Total}(0, A) = 0$

$\text{Single}(1, A) = 1$

$\text{Total}(n, A) = \sum$

$\text{Single}(n, A) =$ smallest value in A, where the gap is larger than $\text{Total}(n-1, A)$ to the next value



$n - 1$