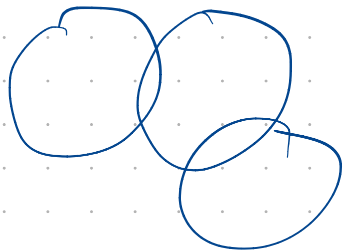
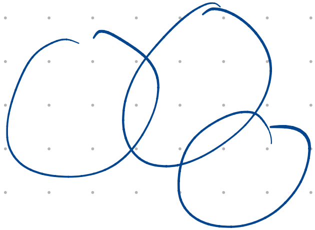
Dynamic Connectivity: Connecting to Networks and Geometry

Chan, Pătraşcu, Roditty '08 IEEE Symposium on Foundations of Computer Science

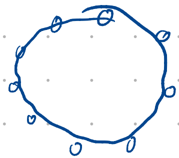




static : easy

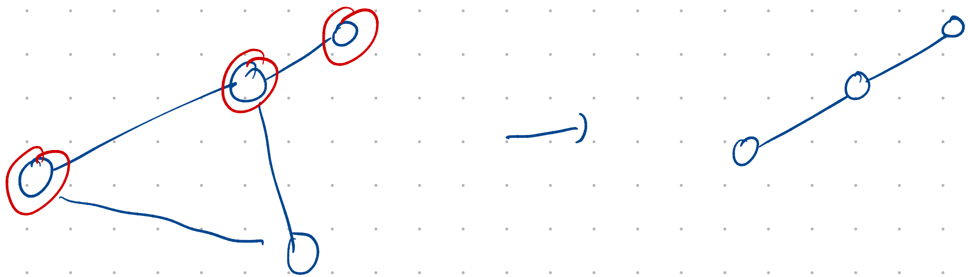
incremental: Union Find

incremental + decremental :

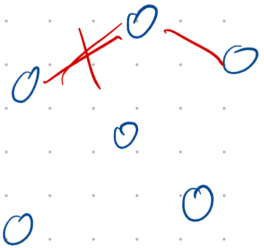


$\tilde{O}(n^{11/12}) \rightsquigarrow$ disks

<u>subgraph connectivity</u>:
- Fixed graph
- single vertices are enabled / disabled
- connectivity in the induced subgraph



Edge update data structures:



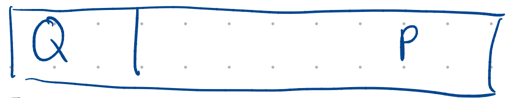
$\rightsquigarrow$ Holm et al. $\rightsquigarrow$ update $O(\log^2 n)$

query $O\left(\frac{\log n}{\log\log n}\right)$

Idea for data structure:

Original graph G

- build auxilliary graph G^*
- split active vertices:



"newly activated"

"old"

- insertions
- deletions

- deletions

after $q = m/\Delta$ updates: merge Q into P

- Split components of P into „high" and „low".

  high: sum of degree of vertices in component $\geq \Delta$

  low: otherwise

high: few $\rightsquigarrow O(m/\Delta)$

low: cheap iterate over outgoing edges.

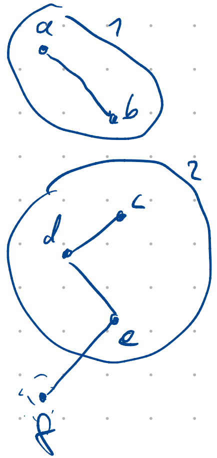
Data structure:

- Maintain F in edge update data structure (e.g. Holm).
- Maintain bipartite multigraph Γ between V and components of F:

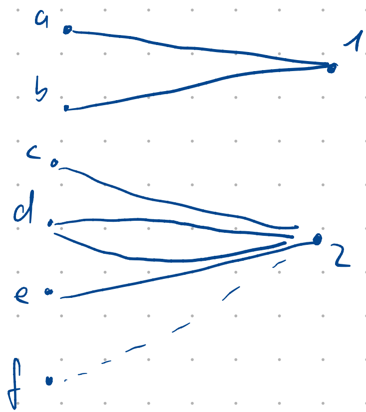
  For $uv \in E$ with v in component γ: Add $u\gamma$ to Γ

example:

G:



Γ:



For $u, v \in V$ maintain $C[u,v]$: # low components, which are adjacent to both u and v

$\Rightarrow O(m\Delta)$ non-zero entries

Now $G^* = (V^*, E^*)$ $\qquad V^* = Q \cup$ components of P

E^*: (a) For each $u, v \in Q$: if $C[u,v] > 0$: $uv \in E^*$

(b) For $u \in Q$, high component γ of P:

if $u\gamma \in \Gamma$: $u\Gamma \in E^*$

(c) For each $u, v \in Q$ if $uv \in E$: $uv \in E^*$

[(d) Both are in P : ✓]

$u, v \in Q$: connected $G \iff$ connected in G^*

$\Leftarrow$ ✓

$\Rightarrow$ direct: (c)
indirect low (a)
high (b)

query: given u, v:

$u, v \in Q$: Test in G^*

$u \in$ high component γ $\nearrow \Gamma$
- find replacement vertex of Q connected to γ if exists.
- if does not exist : test inside component ✓

$u \in$ low component γ;

$\_\_\_$ " $\_\_\_$

$O(1)$

$\Rightarrow \tilde{O}(1)$

updating in Q: $\tilde{O}(m/\Delta)$

$$\left.\begin{array}{l}\text{deletions in high components of } P \\ \phantom{\text{deletions in }}\text{low components of } P\end{array}\right\} \tilde{O}(m\,\Delta)$$

query $\tilde{O}(m^{1/3})$

amortized update: $\tilde{O}(m^{2/3})$

preprocessing $\tilde{O}(m^{4/3})$