On Approximate Range Counting (and Depth)

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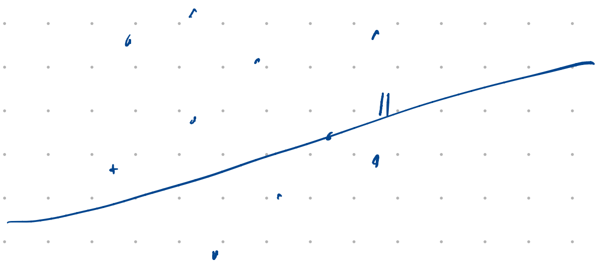
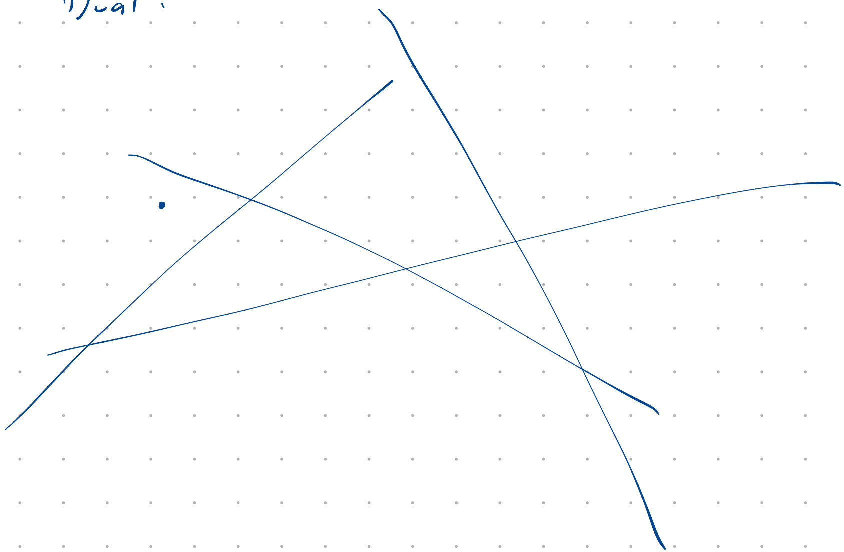
Approximate range counting in $\mathbb{R}^3$ (excl. hyperplanes)



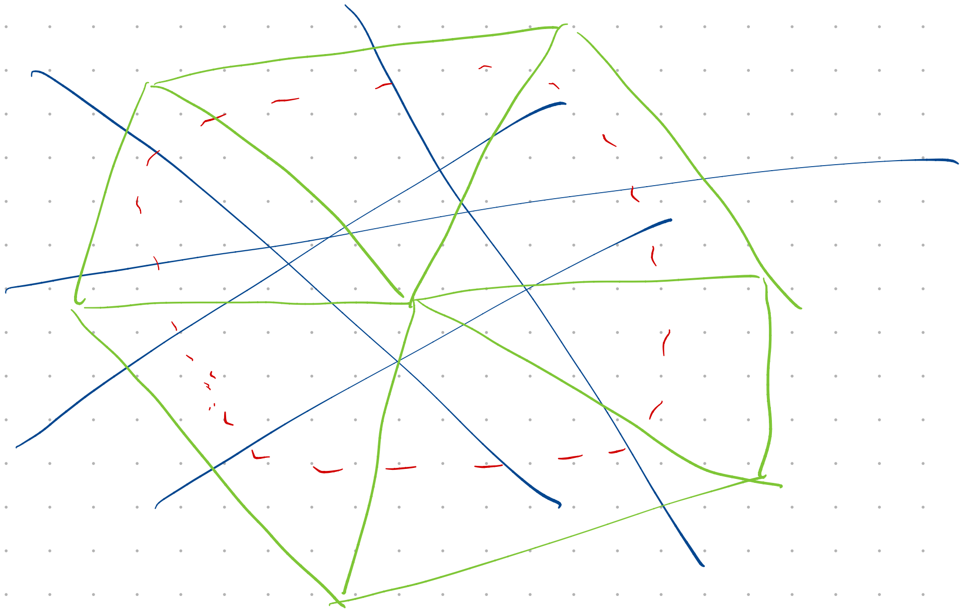
- Reporting: $O(\log n + \underline{\underline{k}})$
- Counting within $(1-\varepsilon)k$ and $(1+\varepsilon)k$
- $O_\varepsilon(n \log n)$ preparation time exp
- $O_\varepsilon(\log n)$ query exp
- $O_\varepsilon(n)$ space exp.

Lemma 4: With $O(n \log n)$ exp. preprocesing we can do a range <u>reporting</u> query in $O(\log n + k)$

Dual:

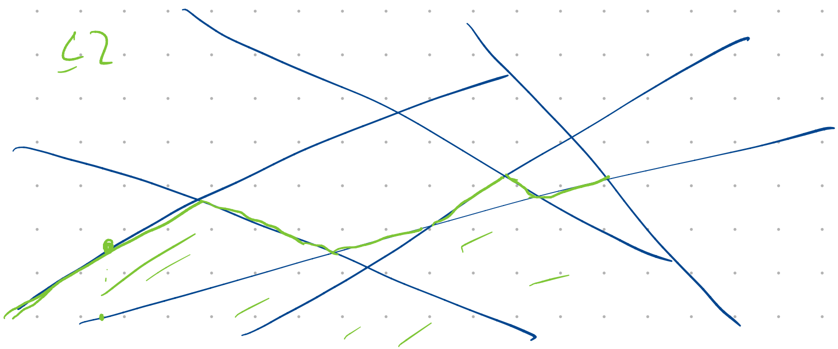


α-cutting



Find cells (tetrahedra) s.th.

- each intersects ≤ αn of the hyperplanes
- All cells together cover the area
- Size of the cutting: # tetrahedra



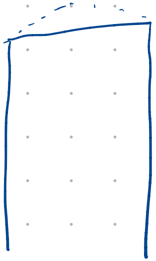
k-level: Exactly k hyperplanes beneath

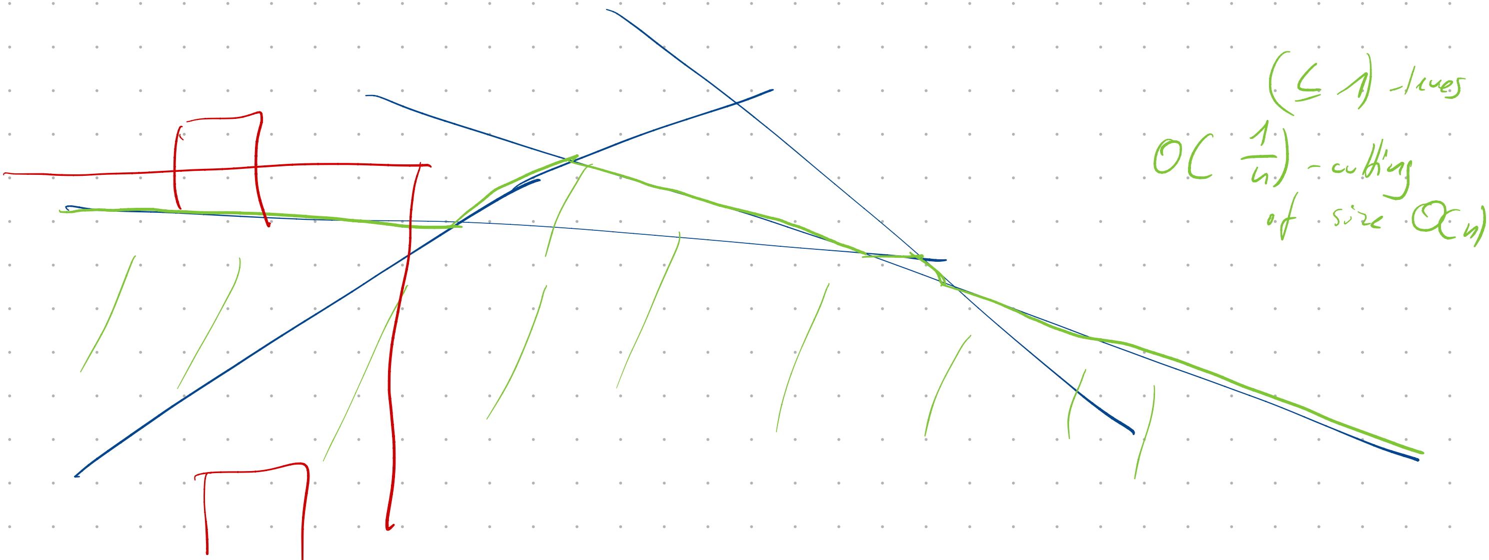
(≤k)-level: All points with ≤k hyperplanes below

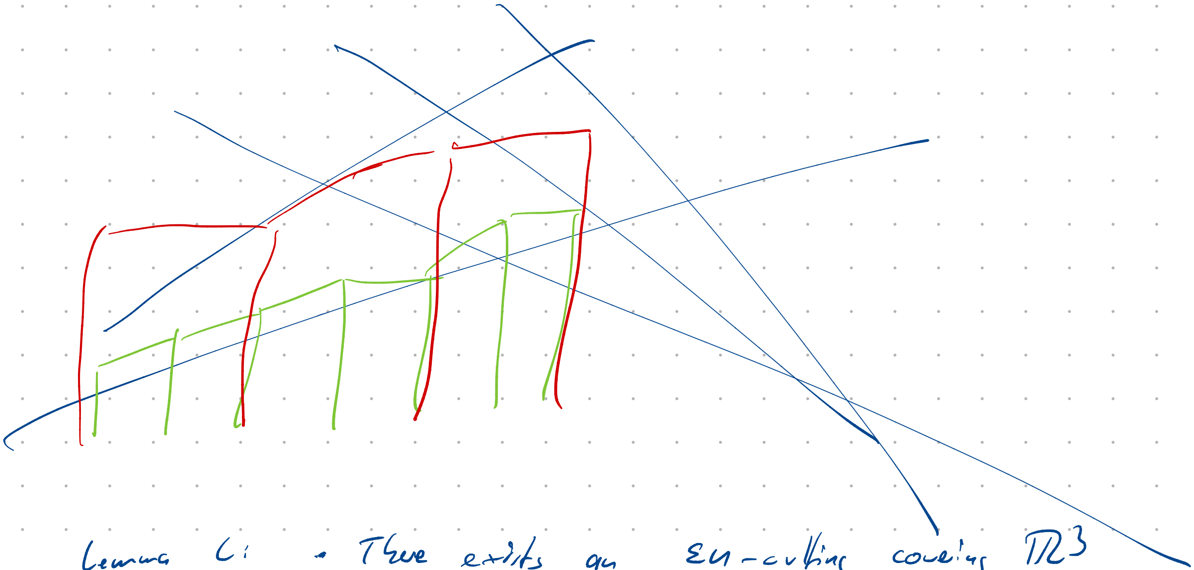
Lemma B:
- There exists an $O(\frac{k}{n})$-cutting of size $O(\frac{n}{k})$ that covers the $(\leq k)$-level

each cell intersects $\geq k$, $\leq k \cdot \alpha$

- The cells are vertical prisms
- sequence of cuttings for all $k = L(1+\varepsilon)^i$) can be constructed together in $O_\varepsilon(n \log n)$ exp. time.



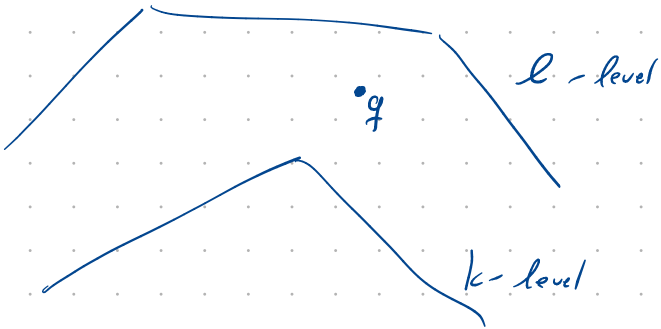




Lemma C: • There exists an εn-cutting covering $\mathbb{R}^3$ of $O_\varepsilon(1)$

• Can construct this cutting $O_\varepsilon(n)$ time

Idea: Use k-levels



$\geq k$ below

$\leq \ell$ below

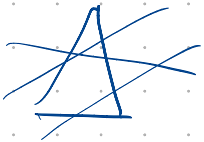
... but approximate:

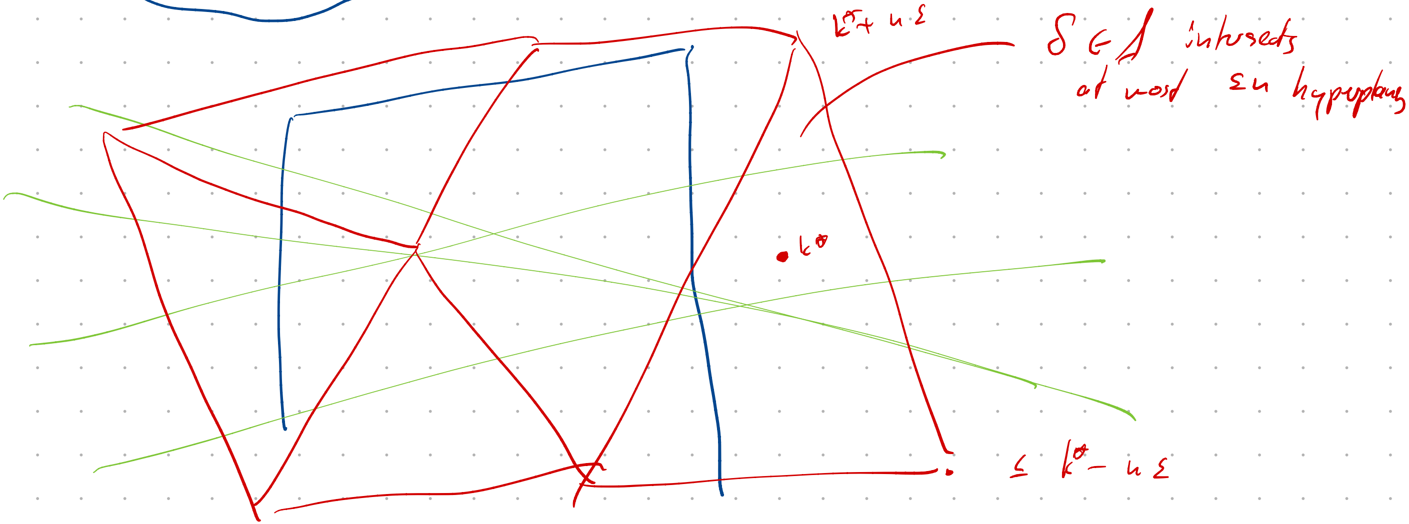
ε-approximate $(\leq k)$-level

- contains $(\leq (1-\varepsilon)k)$-level
- is contained by $(\leq (1+\varepsilon)k)$-level

Data Structure:

- Prepare Lemma A data structure
- Build for each $k = \lfloor (1+\varepsilon)^i \rfloor$ an $O(\frac{k}{n})$ cutting of size $O(\frac{n}{k})$ covering $(\leq k)$-level
- For each prism Δ of some cutting k:
  - We use Lemma A to find all hyperplanes intersecting Δ
  - Use Lemma C to construct an ε-cutting of size $O_\varepsilon(1)$ of Δ with





For each $\delta \in \Delta$:

- Compute level ℓ_δ of δ

$$\frac{1}{\varepsilon^{O(1)}}$$

- If $\ell_\delta \leq k$ then we include δ in $O(\varepsilon)$-approximate cutting

- query: binary search over all cuttings

$O(\log n \cdot \log \log_{1+\varepsilon} n)$